

# Investigation of Plasmonic Properties of Coupled Metal Wires in the Cluster with Triangular or Square Configuration

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**Abstract**—This paper presents a theoretical investigation and a straightforward analysis of the plasmonic properties of coupled metal wires in the cluster with triangular or square configuration. Solution of the eigenvalue problem in form of matrix equations is received. Eigenoscillation frequencies and quality factors of the nanowires of such configurations are studied. Possibility of quality factor enhancement in optimized assemble configurations has been demonstrated.

**Keywords**—Plasma; surface plasmons; plasmon resonances; eigenfrequency; cluster.

## I. INTRODUCTION

Metallic nanostructures are the subject of immense interest in recent years due to the possibility of a strong light localization beyond the diffraction limit via the excitation of surface plasmons (SPs) [1, 2]. Over the last few years, much interest is devoted to metallic nanostructures and, in particular, to the strong electromagnetic enhancement they can provide via the excitations of plasmon resonances [3, 4]. The possibility of using plasmons to create an effective optical nanoantenna [5], subwavelength resonators [6], spacer [7] and to increase the sensitivity of biosensors [8] are widely discussed recently. Surface and localized plasmons have been explored for their potential in single molecule detection [9], biomolecular interaction studies and early stage cancer detection [10]. The problem of interaction of two or more closely spaced wires is of great importance. Such configurations give rise to tunable spectral shifts of the plasmon bands and to exceptionally strong field enhancements.

## II. MATHEMATICAL BACKGROUND: FORMULATION AND SOLUTION

It is known that SPs can exist on a metal wire that can be considered as a plasma infinite-long cylinder (column) in the optical region. In this paper, we consider SPs in the cluster of metal wires with triangular or square configuration. The radius of each wire, embedded in a vacuum, is  $a$ ; the separation

distance between nanowires is  $d$ . The time dependence is  $e^{i\omega t}$ . Figures 1 and 2 represents a schematic diagrams of the cluster with triangular or square configurations respectively. Wires with ordinary nonmagnetic metal is characterised by a negative permittivity  $\varepsilon < 0$ . The frequency dependent plasma permittivity  $\varepsilon_p$  is described by the Drude model

$$\varepsilon_p = 1 - \omega_p^2 \cdot (\omega(\omega - i\gamma))^{-1} \quad (1)$$

where  $\omega_p$  represents the plasma frequency,  $\gamma$  is the material absorption. To describe the fields the polar system of coordinates  $(\rho, \varphi)$  associated with each wire is introduced. The solution is presented in the form of a series of the Bessel functions inside each wire and the second-order Hankel functions in outer medium

$$H(\rho_n, \varphi_n) = \sum_{s=-\infty}^{+\infty} A_s^{(n)} J_s(n_p k \rho_n) e^{is\varphi_n} \quad (2)$$

$$H(\rho_n, \varphi_n) = \sum_{n=1}^N \sum_{s=-\infty}^{+\infty} \bar{A}_s^{(n)} H_s^{(2)}(k \rho_n) e^{is\varphi_n} \quad (3)$$

Here  $k = \omega \cdot c^{-1}$ ,  $n_p = \sqrt{\varepsilon_p}$ ,  $c$  is light velocity in a vacuum,  $N = 3$  or  $N = 4$ .

Unknown coefficients  $A_s$  and  $\bar{A}_s$  are found from the boundary conditions, requiring the continuity of the tangential components of the total electric and magnetic fields at each surface. Using the addition theorem for the Bessel functions we arrive to an infinite system of algebraic equations that can be truncated in order to provide a controlled numerical precision.

The plasmonic properties of wires and particles have recently been investigated using a variety of methods [3, 4, 11-19]. However, there is a lack of investigations in terms of quality ( $Q$ ) factors of SPs, though these characteristics are of

crucial importance in problems associated with spectral resolution of sensors, stimulated emission enhancement, etc. Many authors find SPs investigating resonance peaks in scattering cross section (SCS). This study cannot be considered as a complete one, because in this way only “bright” plasmons can be seen, “dark” plasmons that do not couple efficiently to incident wave cannot be discovered in such a description.

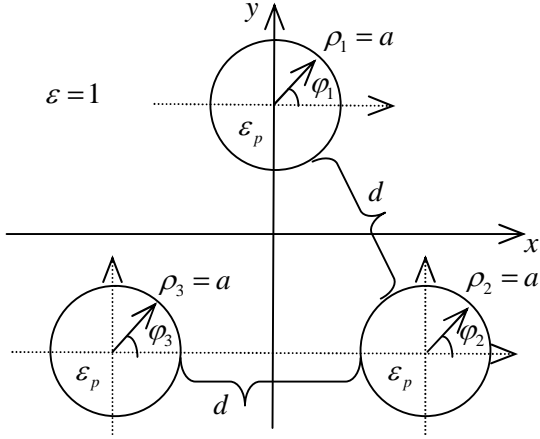


Fig. 1. Schematic diagram of the structure: a cluster of triangular configuration.

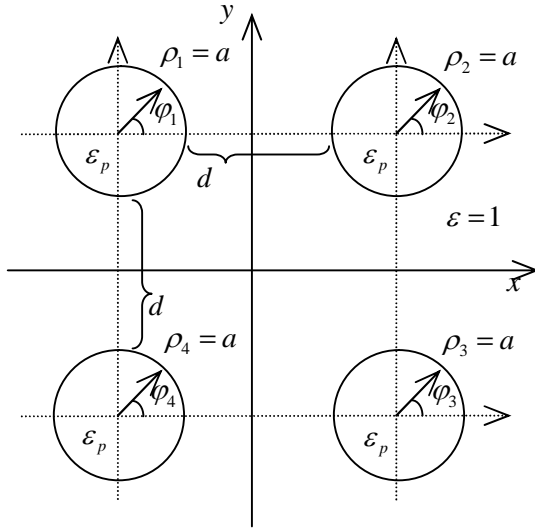


Fig. 2. Schematic diagram of the structure: a cluster of square configuration.

The main goal of this investigation is to develop nonquasistatistical expressions for the eigenvalues of SPs that includes finding of eigenfrequencies and  $Q$ . Using this approach, all possible SPs can be found and investigated, including ‘dark’ and multipole ones.

### III. NUMERICAL RESULTS AND DISSCUSION

For the case of triangular cluster shown in Fig. 1 the structure has three symmetry axes  $x_1, x_2, x_3$  (see Fig. 3); for the case of square cluster shown in Fig. 2 the structure has four symmetry

axes associated with horizontal, vertical, and oblique axes  $x_1, x_2, x_3, x_4$  (see Fig. 4). Similar symmetry classes exist in the photonic molecules of coupled microdisk resonators [20, 21]. Total number of dipole SPs is four for triangular cluster and six for the square cluster [22] (with the same number  $s$  of angular variations of the field). Among the possible excited plasmons there exists e.g. such plasmons with completely symmetrical fields with respect to all the axes of symmetry ( $H^{even}$ , see Fig. 5 (c) and Fig. 6 (c)) and with totally antisymmetrical ones ( $H^{odd}$ , see Fig. 5 (d) and Fig. 6 (d)).

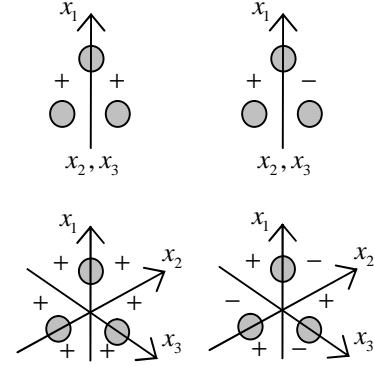


Fig. 3. Classes of symmetry of the field for the cluster with triangular configuration.

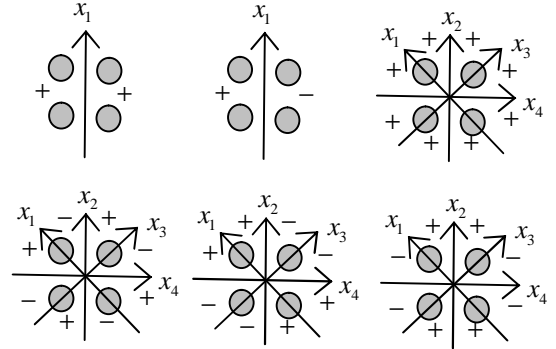


Fig. 4. Classes of symmetry of the field for the cluster with square configuration.

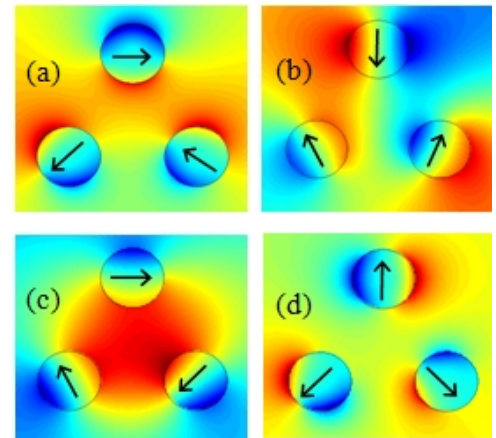


Fig. 5. The near-field distributions of dipole SPs of cluster with triangular configuration ( $d/a = 2$ ).

For modeling results we use the normalized parameter  $w_p = \omega_p a c^{-1}$  that we will call the size parameter and normalized separation distance  $d/a$ .

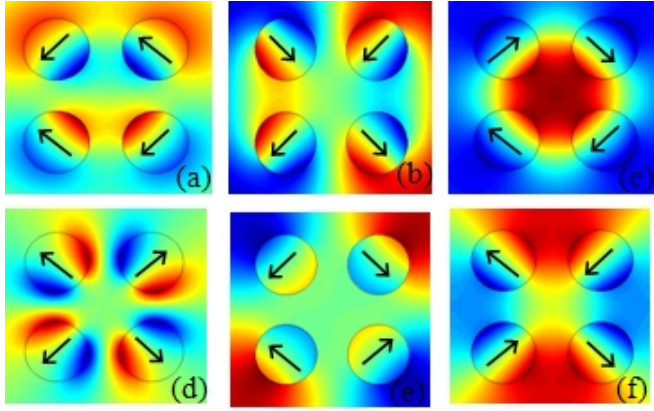


Fig. 6. The near-field distributions of dipole SPs of cluster with square configuration ( $d/a = 1$ ).

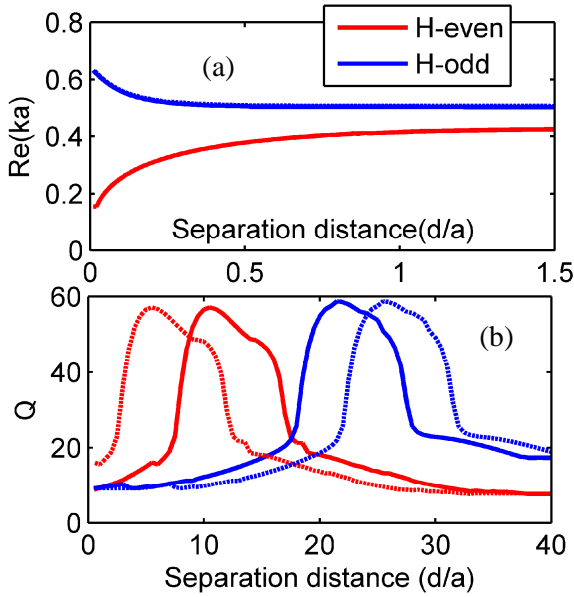


Fig. 7. Dependence of the (a) normalized frequency and (b)  $Q$ -factor on the normalized separation distance between coupled metal wires for the triangular cluster ( $w_p = 1$ ,  $\gamma = w_p \cdot 10^{-3}$ ,  $s = 1$ ).

Figures 5 and 6 show the near-field distributions of dipole SPs of cluster triangular and square configurations respectively ( $w_p = 1$ ). These SPs are symmetric and asymmetric combinations of SPs of individual wire. The orientations of their dipole moments are shown.

All eigenfrequencies are complex  $\omega = \omega' + i\omega''$ , where  $\omega'' > 0$  represents damping and  $\omega'$  is associated with the eigenoscillation frequencies.  $Q$  of plasmons can be evaluated through the formula  $Q = \omega' / 2\omega''$ .

Figure 7 demonstrates the dependence of the normalized frequencies (their real parts) and  $Q$ -factors on the normalized separation distance between coupled metal wires for the triangular cluster ( $w_p = 1$ ,  $\gamma = w_p \cdot 10^{-3}$ ,  $s = 1$ ) for dipole SPs. As normalized separation distance  $d/a$  becomes smaller, the frequency shift of the coupled SPs becomes much stronger.

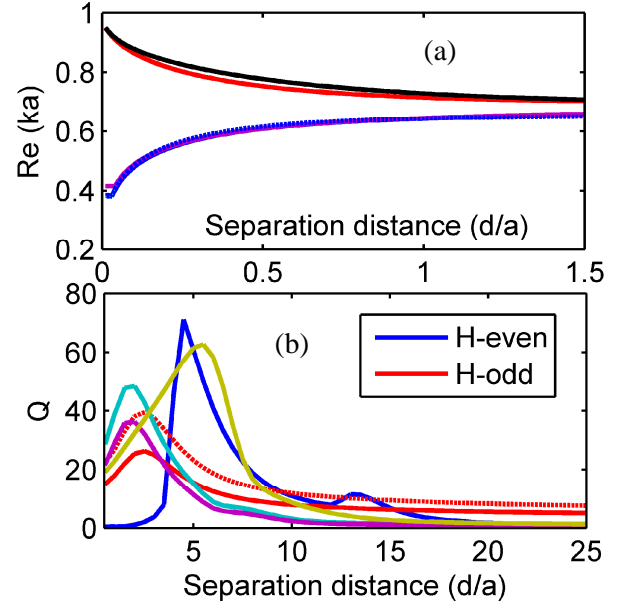


Fig. 8. Dependence of the (a) normalized frequency and (b)  $Q$ -factor on the normalized separation distance between coupled metal wires for the square cluster ( $w_p = 1$ ,  $\gamma = w_p \cdot 10^{-3}$ ,  $s = 1$ ).

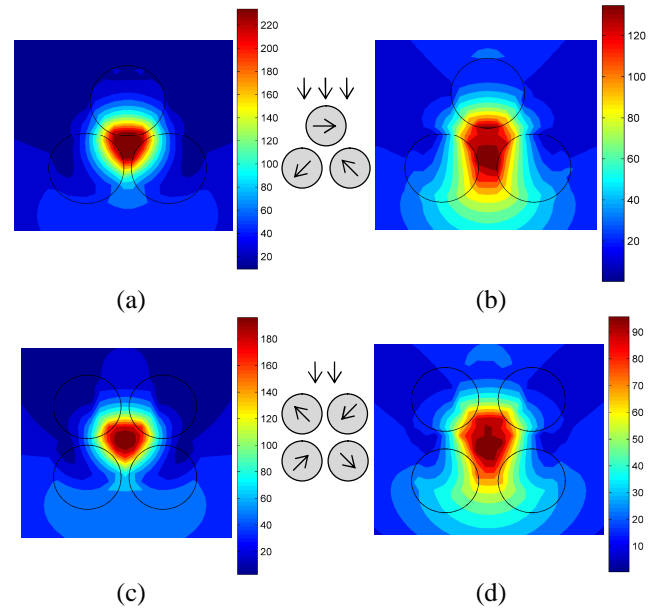


Fig. 9. The near-field distributions of plane wave scattering on coupled metal wires of the cluster triangular or square configuration ( $w_p = 1$ ,  $\gamma = w_p \cdot 10^{-3}$ ): (a), (c)  $d/a = 0.2$ ; (b), (d)  $d/a = 0.5$ .

Dramatical enhancement of  $Q$  is observable when  $d/a=1.1\lambda$  for  $H^{even}$  and  $d/a=2.15\lambda$  for  $H^{odd}$ , where  $\lambda$  is the wavelength.

Figure 8 characterizes the eigenfrequencies (their real parts) and  $Q$ -factors of the all possible dipole SPs in a cluster with square configuration of coupled metal wires. It can be seen that the upward shift in frequency is much faster than downward shift for dipole modes if wires are brought together. Enhancement of  $Q$  is observable when  $d/a=2.26\lambda$  for  $H^{even}$  and  $d/a=2.7\lambda$  for  $H^{odd}$ . Maximum peak of  $Q$ -factor is seen for  $H^{even}$  plasmon.

In a cluster with triangular configuration total number of excited dipole SPs are four: one plasmon is bright (see Fig. 5 (c)) and all other SPs are dark. In a cluster with square configuration total number of excited dipole SPs are six: two plasmons are a bright (see Fig. 6 (c) and (e)) and all other SPs are dark.

Figure 9 shows the near-field distributions of plane wave scattering on coupled metal wires of the cluster triangular and square configurations for different normalized distance: (a) and (c)  $d/a=0.2$ ; (b) and (d)  $d/a=0.5$ . The direction of illumination by plane wave is shown in the inset. When the metal wires are through together the 'hot spots' amplitudes increase.

## CONCLUSIONS

The plasmonic properties of the coupled metal wires arranged in triangular or square cluster have been analyzed. Eigenoscillating frequencies and quality factors of the wires of such configurations are studied. Near fields of hybrid plasmonic modes have been investigated.

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