# Bragg Reflection and Transmission of Light by OneDimensional Gyrotropic Magnetophotonic Crystal 

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#### Abstract

Reflection and transmission coefficients for onedimensional magnetophotonic crystal with a bilayer unit cell are obtained in analytical form. The bilayer consists of arbitrary gyrotropic media. Analytical dispersion equation for gyrotropic multilayer periodic structure is obtained. Different regimes of bulk and surface wave's propagation in structure layers are investigated. System parameters values that correspond to both total reflection and total transmission by gyrotropic Bragg structure are found. The possibility of band structure controlling with an external magnetic field is shown.


Keywords-magnetophotonic one-dimensional crystal; gyrotropic media; ferrite with transversal magnetic field; dispersion characteristics; band gap

## I. Introduction

Photonic crystals allow controlling light properties and shown promising possibilities for development of novel integrated optical devices. Many practical applications of the photonic crystal structures based on their unique spectral properties which are specified by band gap presence [1]. Applications of the electrically and magnetically controllable materials in photonic crystals significantly extend the functionality of the optical devices. Inclusion of liquid crystals and magnetooptical materials provides the additional possibilities of the control the flow of light [2-9].

When the magnetic materials introduced into photonic crystal, the consequent structure is identified as magnetophotonic crystal [10, 11]. Such structures have found many applications in the optical communications and other practical areas. Namely, magnetophotonic crystals are used for development of optical isolator/circulator devices [12], magneto-optic spatial light modulators [13], unidirectional photonic crystal devices [14] etc. Widening of the magnetophotonic crystal applications requires further
experimental and theoretical investigations of the electromagnetic waves interaction with artificial periodic structures that consist of gyrotropic elements.

In this report the scattering of the plane wave on the onedimensional magnetophotonic crystal (Bragg structure) is considered for general case of arbitrary gyrotropy (ferrite or plasma media).

## II. Statement and solution of the problem

In the present work, we study the scattering of a plain Epolarized wave, which is falling on a Bragg gyrotropic reflector. The reflector consists of $N$ periods of bilayer gyrotropic media and additional layer (Fig. 1). We consider that in every region the media differs by the constitutive parameters and geometrical dimensions. As one can see from the figure, there are three regions: region before the reflector, region after the reflector, and the reflector itself. To define the reflection and transmission coefficients of the incident plain wave on the Bragg reflector, one should consider sequentially


Fig. 1. The model of a gyrotropic Bragg structure.
three tasks. In the first task, one needs to find the relation between the field coefficients in the region before the reflector and the field coefficients in the first layer of the Bragg reflector. In the second task, one needs to find the relation between the first and the last layer of the Bragg structure. Finally, in the third task, one needs to find the relation between the last layer of the reflector and the field coefficients after the reflector.

We study wave's propagation in a stratified bilayer periodic structure with gyrotropic layers (one-dimensional magnetophotonic crystal) (Fig. 1). Every of two layers on the structure period $l=a+b$ is anisotropic media (plasma or ferrite or their combinations) with dielectric and magnetic permeabilities characterized by tensor values of standard form with constitutive parameters $\vec{\varepsilon}_{j}, \vec{\mu}_{j}(j=1,2)$.

The tensors of dielectric $\vec{\varepsilon}_{j}$ and magnetic $\vec{\mu}_{j}$ permeabilities have the following form [15]:

$$
\stackrel{\leftrightarrow}{\varepsilon_{j}}=\left\|\begin{array}{ccc}
\varepsilon_{j} & -i \varepsilon_{a j} & 0  \tag{1}\\
i \varepsilon_{a j} & \varepsilon_{j} & 0 \\
0 & 0 & 1
\end{array}\right\|, \stackrel{\leftrightarrow}{\mu_{j}}=\left\|\begin{array}{ccc}
\mu_{j} & -i \mu_{a j} & 0 \\
i \mu_{a j} & \mu_{j} & 0 \\
0 & 0 & 1
\end{array}\right\| .
$$

For the plasma medium, the value of the permittivity $\vec{\varepsilon}_{j}$ is tensor, while the value of the magnetic permeability $\mu_{j}$ is scalar. Such media are called electrically-gyrotropic. In case of ferrite medium, the other way around, the value of the magnetic permeability $\vec{\mu}_{j}$ is tensor, while the permittivity $\varepsilon$ is scalar. Such media are called magnetically-gyrotropic. If the dielectric and magnetic permeabilities of the media are both described by the tensors (1), then such media is called gyrotropic. The constitutive parameters, which are included in the tensors of dielectric $\vec{\varepsilon}_{j}$ and magnetic $\vec{\mu}_{j}$ permeabilities, are defined by the value of the external controlling magnetic field $\vec{H}_{0}=\vec{z}_{0} H_{0}$. The study of the general case of gyrotropic media with constitutive parameters (1) is reasonable, first of all, because it enables us to use the principle of permutation duality [15], when relations $\vec{E} \leftrightarrow \vec{H}, \vec{\varepsilon} \leftrightarrow-\vec{\mu}$ are satisfied, to obtain main equation for fields and characteristic equations. It follows directly from the Maxwell equations.

It is well known [15], that there are two independent Maxwell equations solutions, two types of waves $-H_{z}$ (TE) waves and $E_{z}(\mathrm{TM})$ waves in case of two-dimensional gyrotropic media $(\partial / \partial z=0)$.

The electromagnetic waves for TE waves ( $H_{z}-$ polarization) and TM waves ( $E_{z}$ - polarization) are described by two independent solutions of Helmholtz equation, obtained directly from Maxwell equations:

$$
\begin{equation*}
\frac{\partial^{2} H_{z}}{\partial x^{2}}+\frac{\partial^{2} H_{z}}{\partial y^{2}}+k^{2} \varepsilon_{\perp_{j}} \mu_{j} H_{z}=0 \tag{2}
\end{equation*}
$$

$$
\begin{align*}
& E_{y}=\left(\frac{1}{i k \varepsilon_{\perp j}}\right)\left(\frac{\partial H_{z}}{\partial x}+i \frac{\varepsilon_{a j}}{\varepsilon_{j}} \frac{\partial H_{z}}{\partial y}\right) .  \tag{3}\\
& \frac{\partial^{2} E_{z}}{\partial x^{2}}+\frac{\partial^{2} E_{z}}{\partial y^{2}}+k^{2} \mu_{\perp j} \varepsilon_{j} E_{z}=0 .  \tag{4}\\
& H_{y}=-\left(\frac{1}{i k \mu_{\perp j}}\right)\left(\frac{\partial E_{z}}{\partial x}+i \frac{\mu_{a j}}{\mu_{j}} \frac{\partial E_{z}}{\partial y}\right),  \tag{5}\\
& \mu_{\perp j}=\mu_{j}\left(1-\frac{\mu_{a j}^{2}}{\mu_{j}^{2}}\right), \varepsilon_{\perp j}=\varepsilon_{j}\left(1-\frac{\varepsilon_{a j}^{2}}{\varepsilon_{j}^{2}}\right) \tag{6}
\end{align*}
$$

Relations (3) and (5) are obtained from Maxwell equations taking into account the tensor form (1) of magnetic $\vec{\mu}_{j}$ and electric $\vec{\varepsilon}_{j}$ permeabilities. They are used for finding the tangential components of electromagnetic field in every layer of the gyrotropic periodic media.

As one can simply notice, the Helmholtz solutions (2), (4) are interchangeable, if we replace $H_{z}$ with $E_{z}$ and correspondently replace of all the permittivities $\varepsilon$ with the magnetic permeabilities $(-\mu)$, and vice-versa. So taking into account the principle of permutation duality for fields, the further investigation can be done only for one of the described above types of waves. Here we describe only $E_{z}$ polarization (TM waves). For TE waves we use the principle of permutation duality.

## A. The first task

To complete the first task, we write solution of the Helmholtz equation (4) for the region before the multilayer structure and the first layer of the Bragg reflector:

$$
\begin{gather*}
E_{z}^{0}(x, y)=\left(a^{0} e^{i \xi_{0} x}+b^{0} e^{-i \xi_{0} x}\right) e^{i \beta y}, \\
H_{y}^{0}(x, y)=\frac{\xi_{0}}{-k \mu_{0}}\left(a^{0} e^{i \xi_{0} x}-b^{0} e^{-i \xi_{0} x}\right) e^{i \beta y},  \tag{7}\\
E_{z}^{1}(x, y)=\left(a_{0} e^{i \xi_{1} x}+b_{0} e^{-i \xi_{1} x}\right) e^{i \beta y}, \\
H_{y}^{1}(x, y)=\frac{1}{-k \mu_{\perp 1}}\binom{\left.a_{0}\left(\xi_{1}+i \frac{\mu_{a 1}}{\mu_{1}} \beta\right) e^{i \xi_{1} x}-\right)}{-b_{0}\left(\xi_{1}-i \frac{\mu_{a 1}}{\mu_{1}} \beta\right) e^{-i \xi_{1} x}} e^{i \beta y}, \tag{8}
\end{gather*}
$$

The boundary conditions for tangential components lead to the following matrix equation:

$$
\left(\begin{array}{cc}
1 & 1  \tag{9}\\
1 & -1
\end{array}\right)\binom{a^{0}}{b^{0}}=V\binom{a_{0}}{b_{0}}
$$

$V=\left(\begin{array}{c}e^{-i \xi_{1} a} \\ \frac{\xi_{1}}{\mu_{\perp 1}} \frac{\mu_{\perp 0}}{\xi_{0}}\left(1+\frac{i \mu_{a 1}}{\mu_{1}} \frac{\beta}{\xi_{1}}\right) e^{-i \xi_{1} a} \\ -\frac{\xi_{1}}{\mu_{\perp 1}} \frac{\mu_{\perp 0}}{\xi_{0}}\left(1-\frac{i \mu_{a 1}}{\mu_{1}} \frac{\beta}{\xi_{1}}\right) e^{i \xi_{1} a}\end{array}\right)$
Using the inverse matrix and the rule of matrices multiplication, one can obtain the following matrix equation for column vector $\binom{a^{0}}{b^{0}}$ :

$$
\binom{a^{0}}{b^{0}}=\left(\begin{array}{ll}
n_{11} & n_{12}  \tag{10}\\
n_{21} & n_{22}
\end{array}\right)\binom{a_{0}}{b_{0}}
$$

where matrix elements $n_{i j}$ are defined by the expressions obtained by matrices multiplication according to the definition

$$
\begin{align*}
& n_{11}=\frac{1}{2}\left[1+\frac{\xi_{1}}{\mu_{\perp 1}} \frac{\mu_{0}}{\xi_{0}}\left(1+i \frac{\mu_{a 1}}{\mu_{1}} \frac{\beta}{\xi_{1}}\right)\right] e^{-i \xi_{1} a},  \tag{11}\\
& n_{12}=\frac{1}{2}\left[1-\frac{\xi_{1}}{\mu_{\perp 1}} \frac{\mu_{0}}{\xi_{0}}\left(1-i \frac{\mu_{a 1}}{\mu_{1}} \frac{\beta}{\xi_{1}}\right)\right] e^{i \xi_{1} a},  \tag{12}\\
& n_{21}=\frac{1}{2}\left[1-\frac{\xi_{1}}{\mu_{\perp 1}} \frac{\mu_{0}}{\xi_{0}}\left(1+i \frac{\mu_{a 1}}{\mu_{1}} \frac{\beta}{\xi_{1}}\right)\right] e^{-i \xi_{1} a},  \tag{13}\\
& n_{22}=\frac{1}{2}\left[1+\frac{\xi_{1}}{\mu_{\perp 1}} \frac{\mu_{0}}{\xi_{0}}\left(1-i \frac{\mu_{a 1}}{\mu_{1}} \frac{\beta}{\xi_{1}}\right)\right] e^{i \xi_{5} a} . \tag{14}
\end{align*}
$$

## B. The second task

Using obtained results, one can connect unknown coefficients for electromagnetic fields in the first and the last layers of the reflector, namely

$$
\begin{align*}
\binom{a_{0}}{b_{0}} & =\left(\begin{array}{ll}
A & B \\
C & D
\end{array}\right)\binom{a_{1}}{b_{1}}=\left(\begin{array}{ll}
A & B \\
C & D
\end{array}\right)\left(\begin{array}{ll}
A & B \\
C & D
\end{array}\right)\binom{a_{2}}{b_{2}}=\ldots  \tag{15}\\
& =\left(\begin{array}{ll}
A & B \\
C & D
\end{array}\right)^{n}\binom{a_{n}}{b_{n}}=\left(\begin{array}{ll}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{array}\right)\binom{a_{n}}{b_{n}} .
\end{align*}
$$

Where

$$
M=\left(\begin{array}{ll}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{array}\right)=\left(\begin{array}{ll}
A & B \\
C & D
\end{array}\right)^{n}
$$

is unimodular matrix. For unimodular matrices the following identity should be satisfied:

$$
M=\left(\begin{array}{cc}
A U_{N-1}-U_{N-2} & B U_{N-1}  \tag{16}\\
C U_{N-1} & D U_{N-1}-U_{N-2}
\end{array}\right)
$$

where

$$
\begin{gather*}
U_{N}=\frac{\sin (N+1) K \Lambda}{\sin K \Lambda}, M_{11}=A \frac{\sin (N) K \Lambda}{\sin K \Lambda}-\frac{\sin (N-1) K \Lambda}{\sin K \Lambda}, \\
M_{12}=B U_{N-1}=B \frac{\sin (N) K \Lambda}{\sin K \Lambda}, M_{21}=C U_{N-1}=C \frac{\sin (N) K \Lambda}{\sin K \Lambda}, \\
M_{22}=D \frac{\sin (N) K \Lambda}{\sin K \Lambda}-\frac{\sin (N-1) K \Lambda}{\sin K \Lambda}, \tag{17}
\end{gather*}
$$

and $K$ is Bloch wave number which is defined by the characteristic equation for infinite periodical structure [16,17], namely:
$K(\beta)=\frac{1}{l} \arccos \left\{\cos \xi_{2} b \cos \xi_{1} a-\frac{1}{2}\left[\begin{array}{l}\frac{\xi_{1}}{\xi_{2}} \frac{\mu_{\perp 2}}{\xi_{\perp 1}}+\frac{\xi_{2}}{\xi_{1}} \frac{\mu_{\perp 1}}{\mu_{\perp 2}}+ \\ +\frac{\beta^{2}}{\xi_{1} \xi_{2}} \frac{\mu_{\perp 2}}{\mu_{\Perp 1}}\left(\frac{\mu_{a 1}}{\mu_{1}}-\frac{\mu_{\Perp 1}}{\mu_{\perp 2}} \frac{\mu_{a 2}}{\mu_{2}}\right)^{2}\end{array}\right] \sin \xi_{2} b \sin \xi_{1} a\right\}$
Elements of the transmission matrix $A B C D$ for the periodic gyromagnetic structure have the following form:

$$
\begin{aligned}
& A=\left\{\cos \xi_{2} b-i \frac{1}{2}\left[\frac{\xi_{1}}{\xi_{2}} \frac{\mu_{\perp 2}}{\mu_{\perp 1}}+\frac{\xi_{2}}{\xi_{1}} \frac{\mu_{\perp 1}}{\mu_{\perp 2}}+\frac{\beta^{2}}{\xi_{1} \xi_{2}} \frac{\mu_{\perp 2}}{\mu_{\perp 1}}\left(\frac{\mu_{a 1}}{\mu_{1}}-\frac{\mu_{\perp 1}}{\mu_{\perp 2}} \frac{\mu_{a 2}}{\mu_{2}}\right)^{2}\right] \sin \xi_{2} b\right\} e^{-i \xi_{1} a} \\
& D=\left\{\cos \xi_{2} b+i \frac{1}{2}\left[\frac{\xi_{1}}{\xi_{2}} \frac{\mu_{\perp 2}}{\mu_{\perp 1}}+\frac{\xi_{2}}{\xi_{1}} \frac{\mu_{\perp 1}}{\mu_{\perp 2}}+\frac{\beta^{2}}{\xi_{1} \xi_{2}} \frac{\mu_{\perp 2}}{\mu_{\perp 1}}\left(\frac{\mu_{a 1}}{\mu_{1}}-\frac{\mu_{\perp 1}}{\mu_{\perp 2}} \frac{\mu_{a 2}}{\mu_{2}}\right)^{2}\right] \sin \xi_{2} b\right\} e^{i \xi_{1} a} \\
& B=i \frac{1}{2} \sin \xi_{2} b\left\{-\frac{\xi_{2}}{\xi_{1}} \frac{\mu_{\perp 1}}{\mu_{\perp 2}}+\frac{\xi_{1}}{\xi_{2}} \frac{\mu_{\perp 2}}{\mu_{\perp 1}}\left[1-i \frac{\beta}{\xi_{1}}\left(\frac{\mu_{a 1}}{\mu_{1}}-\frac{\mu_{a 2}}{\mu_{2}} \frac{\mu_{\perp 1}}{\mu_{\perp 2}}\right)\right]^{2}\right\} e^{i \xi_{1} a} \\
& C=-i \frac{1}{2} \sin \xi_{2} b\left\{-\frac{\xi_{2}}{\xi_{1}} \frac{\mu_{\perp 1}}{\mu_{\perp 2}}+\frac{\xi_{1}}{\xi_{2}} \frac{\mu_{\perp 2}}{\mu_{\perp 1}}\left[1+i \frac{\beta}{\xi_{1}}\left(\frac{\mu_{a 1}}{\mu_{1}}-\frac{\mu_{a 2}}{\mu_{2}} \frac{\mu_{\perp 1}}{\mu_{\perp 2}}\right)\right]^{2}\right\} e^{-i \xi_{1} a}
\end{aligned}
$$

The important property of $A B C D$ matrix is the unimodularity, when between the elements of matrix we have equality: $A D-B C=1$. Using the expressions for matrix elements $A B C D$ one can show that this condition is satisfied. Note, that when $\xi_{1}^{2}=k^{2} \varepsilon_{1} \mu_{\perp 1}-\beta^{2}>0, \xi_{1}$ is a real number, $A=D^{*}$ and $B=C^{*}$ as follows from the matrix elements.

Elements of matrix $M$ can be calculated numerically by multiplication of matrix $A B C D$ or analytically from Chebyshev polynomial $U_{N}$. Using the property of inversed matrices one can also find:

$$
\binom{a_{n}}{b_{n}}=\left[\left(\begin{array}{ll}
A & B  \tag{18}\\
C & D
\end{array}\right)^{-1}\right]^{n}\binom{a_{0}}{b_{0}}=\left(\begin{array}{cc}
D & -B \\
-C & A
\end{array}\right)^{n}\binom{a_{0}}{b_{0}}
$$

If the number of periods is fixed ( $N$ ), then in the equations we need to replace index $n$ with $N$.

## C. The third task

Here we study the task of the field scattering of the interface of the reflector last layer with the external media.

The electric and magnetic fields in the last layer of the gyrotropic Bragg structure:

$$
\begin{align*}
& E_{z}^{N}(x, y)=\left(a_{N} e^{i \xi_{1}(x-N l)}+b_{N} e^{-i \xi_{1}(x-N l)}\right) e^{i \beta y}, \\
& H_{y}^{N}(x, y)=\frac{1}{-k \mu_{\perp 1}}\left(\begin{array}{l}
a_{N}\left(\xi_{1}+i \frac{\mu_{a 1}}{\mu_{1}} \beta\right) e^{i \xi_{1}(x-N l)}- \\
\left.-b_{N}\left(\xi_{1}-i \frac{\mu_{a 1}}{\mu_{1}} \beta\right) e^{-i \xi_{1}(x-N l)}\right) e^{i \beta y} .
\end{array} .\right. \tag{19}
\end{align*}
$$

We present field in the media after the Bragg reflector in the following way:

$$
\begin{align*}
& E_{z}^{3}(x, y)=\left(a_{3} e^{i \xi_{3}(x-N l)}+b_{3} e^{-i \xi_{3}(x-N l)}\right) e^{i \beta y}, \\
& H_{y}^{3}(x, y)=\frac{\xi_{3}}{-k \mu_{3}}\left(a_{3} e^{i \xi_{3}(x-N l)}-b_{3} e^{-i \xi_{3}(x-N l)}\right) e^{i \beta y} . \tag{20}
\end{align*}
$$

The boundary conditions for the tangential components of the field lead to the following matrix equations for unknown field coefficients in this region:

$$
\begin{array}{r}
\left(\left(1+i \frac{\mu_{a 1}}{\mu_{1}} \frac{\beta}{\xi_{1}}\right)-\left(1-i \frac{\mu_{a 1}}{\mu_{1}} \frac{\beta}{\xi_{1}}\right)\binom{a_{N}}{b_{N}}=\right.  \tag{21}\\
=\left(\begin{array}{cc}
1 & 1 \\
\frac{\xi_{3}}{\mu_{3}} \frac{\mu_{\Perp 1}}{\xi_{1}} & -\frac{\xi_{3}}{\mu_{3}} \frac{\mu_{\Perp 1}}{\xi_{1}}
\end{array}\right)\binom{a_{3}}{b_{3}}
\end{array}
$$

This equation can be transformed into matrix equation of a standard type:

$$
\binom{a_{N}}{b_{N}}=\left(\begin{array}{ll}
k_{11} & k_{12}  \tag{22}\\
k_{21} & k_{22}
\end{array}\right)\binom{a_{3}}{b_{3}},
$$

where

$$
\begin{aligned}
& k_{11}=\frac{1}{2}\left[\left(1-i \frac{\mu_{a 1}}{\mu_{1}} \frac{\beta}{\xi_{1}}\right)+\frac{\mu_{\perp 1}}{\xi_{1}} \frac{\xi_{3}}{\mu_{3}}\right], \\
& k_{12}=\frac{1}{2}\left(\left(1-i \frac{\mu_{a 1}}{\mu_{1}} \frac{\beta}{\xi_{1}}\right)-\frac{\mu_{\perp 1}}{\xi_{1}} \frac{\xi_{3}}{\mu_{3}}\right), \\
& k_{21}=\frac{1}{2}\left(\left(1+i \frac{\mu_{a 1}}{\mu_{1}} \frac{\beta}{\xi_{1}}\right)-\frac{\mu_{\perp 1}}{\xi_{1}} \frac{\xi_{3}}{\mu_{3}}\right), \\
& k_{22}=\frac{1}{2}\left(\left(1+i \frac{\mu_{a 1}}{\mu_{1}} \frac{\beta}{\xi_{1}}\right)+\frac{\mu_{\perp 1}}{\xi_{1}} \frac{\xi_{3}}{\mu_{3}}\right) .
\end{aligned}
$$

Since in media « 3 » after the reflector, there are no reflected waves, then $b_{3}=0$ and equation (22) takes the form:

$$
\binom{a_{N}}{b_{N}}=\left(\begin{array}{ll}
k_{11} & k_{12}  \tag{23}\\
k_{21} & k_{22}
\end{array}\right)\binom{a_{3}}{0}=\binom{k_{11}}{k_{21}} a_{3},
$$

Finally one can write matrix equation, which represents the scattering of initial plain wave on the Bragg reflector, using equations (10), (15) and (23), namely

$$
\binom{a^{0}}{b^{0}}=\left(\begin{array}{ll}
n_{11} & n_{12}  \tag{24}\\
n_{21} & n_{22}
\end{array}\right)\left(\begin{array}{ll}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{array}\right)\binom{k_{11}}{k_{21}} a_{3}
$$

From this matrix equation one can simply find the reflection and transmission coefficients for a plain wave incident on the Bragg reflector from the media « $0 »$, namely:
$R=\frac{b^{0}}{a^{0}}=\frac{\left[\left(n_{21} M_{11}+n_{22} M_{21}\right) k_{11}+\left(n_{21} M_{12}+n_{22} M_{22}\right) k_{21}\right]}{\left[\left(n_{11} M_{11}+n_{12} M_{21}\right) k_{11}+\left(n_{11} M_{12}+n_{12} M_{22}\right) k_{21}\right]}$,
$T=\frac{a_{3}}{a^{0}}=\frac{1}{\left[\left(n_{11} M_{11}+n_{12} M_{21}\right) k_{11}+\left(n_{11} M_{12}+n_{12} M_{22}\right) k_{21}\right]}$.

The obtained expressions can be used both for electricallygyrotropic and magnetically-gyrotropic media.

## III. ANALYSIS OF RESULTS

As the analysis shows, for such structures there are transmission and forbidden bands, which can be controlled by the applied magnetic field. A number of variants of dispersion curves are found. Depending on the problem parameters, in the structure one can find both fast bulk waves and slow surface waves on the layers interfaces, and also their combinations in every single layer. The solutions for fast waves are defined by the conditions $k^{2} \varepsilon_{2} \mu_{2}>\beta^{2}$ and $k^{2} \varepsilon_{1} \mu_{1}>\beta^{2}$ for corresponding layer. To define conditions for slow waves, one should reverse the sings in the inequalities into the opposite.

Fig. 2 shows an example of dispersion diagrams - the projection of function $K(k, \beta)$ on the plane $(k, \beta)$ in threedimensional wave number space. The shadowed regions correspond to the transmission bands, while light regions correspond to the forbidden bands. Fig. 2 shows dispersion characteristics of magnetophotonic crystal, when both layers on the period are anisotropic ferrite media. As one can notice,


Fig. 2. Dispersion characteristics of a magnetophotonic crystal with two ferrite layers.


Fig. 3. The dependence of reflection coefficient modulus on the frequency parameter.


Fig. 4. The dependence of transmission coefficient modulus on the frequency parameter.
in this case with different width of each of the layers one can considerably change the width of the forbidden region and control its position in the dispersion diagram.

Fig. 3 and Fig. 4 show spectral dependencies of reflection and transmission coefficients for Bragg ferrite crystal. In this case structure consists five periods. Calculations are performed for such parameters: $\varepsilon_{1}=\varepsilon_{2}=1 ; \quad \mu_{1}=1.5$; $\mu_{a 1}=1.0 ; \mu_{2}=4 ; \mu_{a 2}=5.18$. The analysis shows that there are frequency regions of total transmission and reflection. Their width and position can be controlled with de magnetic field.

## IV. Conclusions

The analytical expressions for reflection and transmission coefficients of a plain wave in the one-dimensional magnetophotonic crystal are found. The dispersion equation
and its solutions are obtained. It is shown that there are different regimes - fast bulk waves, slow bulk and surface waves, and their combinations depending on the problem parameters. In addition, we found frequency regions of total transmission and reflection on the spectral characteristics. Width and position of these regions can be controlled by means of the external dc magnetic field.

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