SYSTEM SPECTRAL ANALYSIS OF THE ULTRAWIDEBAND SIGNALS

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Abstract

The new integrated signal analysis method called as system spectral analysis and based on the simultaneous application of the set of linear and non-linear integral transforms is proposed and realized. The abilities of the system spectral analysis at the sample of investigation of the terahertz electromagnetic pulse considered as ultrawideband (UWB) signal are shown.

Keywords: Digital signal processing, wavelet transforms, adaptive Fourier transform, Cohen class non-linear transforms, UWB signals and processes.

1. INTRODUCTION

The creation and the more frequent usage of new signal types, such as ultra-short [1], direct-chaotic [2], non-linear [12], fractal ultrawideband (UWB) [3], fractal [4] ones, in different branches of science and engineering calls for the development of new mathematical methods of signal analysis allowing to detect and to describe the features of such signals at higher level comparing with traditional Fourier transform and their modifications.

Different types of wavelet transform [5], atomic functions [6], adaptive Fourier transform [7], non-linear transforms from Cohen's class, in particular, Wigner and Choi-Williams transforms [8] were made a good showing during the analysis of such signals [9, 10, 13-20].

Nevertheless, each transform from that has both the unique properties and some disadvantages. Therefore, the creation of new integrated method of signal analysis, in which the disadvantages of ones transforms will be compensated by the advantages of other transforms, is appeared to be advisable and topical.

2. SYSTEM SPECTRAL ANALYSIS BASES

The proposition of creation of the new method based on the simultaneous usage of transform set and called as the system spectral analysis seems to be modern and useful.

This set covers two different groups of continuous transforms – linear and non-linear transforms. In first group there are the continuous wavelet transform (CWT), the analytical wavelet transform (AWT), the Gabor transform (GT), the adaptive Fourier transform (AFT) and the short-time Fourier transform (STFT). Second group includes the Fourier spectrogram (FS), the Wigner transform (WiT), the Choi-Williams transform (ChWT) and the Born-Jordan transform BJT. Let's consider these groups in detail.

2.1. LINEAR TRANSFORMS

The continuous wavelet transform [5] is given by

$$Wf(T,\tau) = |kT|^{-1/2} \int\limits_{-\infty}^{\infty} s(t)\psi\left(\frac{t-\tau}{kT}\right)dt\,,$$

where $Wf(T,\tau)$ is the CWT spectral density function (SDF), $s(t)$ is the analyzed signal, $\psi(t)$ is the analyzing real wavelet, τ is the time-shift variable, T is time-scale variable, connected with ordinary scale variable a by the relation $a = kT$, where k is the factor, determined for each wavelet $\psi(t)$.

The analytical wavelet transform [5] is given by

$$\dot{W}f(T,\tau) = |kT|^{-1/2} \int\limits_{-\infty}^{\infty} s(t)\psi^*\left(\frac{t-\tau}{kT}\right)dt\,,$$

where $\dot{W}f(T,\tau)$ is the AWT SDF, $\psi^*(t)$ is the complex conjugation of the complex wavelet function $\dot{\psi}(t)$.

The Gabor transform [5] is given by

$$\dot{G}f(T,\tau) = \frac{1}{(\pi\sigma^2)^{1/4}} \int\limits_{-\infty}^{\infty} s(t) \times$$
$$\times \exp\left(-\frac{(t-\tau)^2}{2\sigma^2}\right)\exp\left(-i2\pi\frac{t}{T}\right)dt,$$

where $\dot{G}f(T,\tau)$ is the GT SDF, σ is the half-width of the Gauss' spectral window.

The adaptive Fourier transform [7] is given by

$$\dot{A}_\nu f(T,\tau) = \frac{1}{\sqrt{T}} \int_{-\infty}^{\infty} s(t) \times \\ \times g\left(\frac{t-\tau}{T}\right) \exp\left[-i\pi\nu\left(\frac{t-\tau}{T}\right)\right] dt,$$

where $\dot{A}_\nu f(T,\tau)$ is the complex-valued AFT SDF, $g(t)$ is the window function, ν is positive coefficient which is equal to the number of harmonic function periods covered in the window $g(t)$.

The short-time Fourier transform [5] is given by

$$\dot{S}f(T,\tau) = \int_{-\infty}^{\infty} s(t) g(t-\tau) \exp\left(-i2\pi \frac{t}{T}\right) dt,$$

where $\dot{S}f(T,\tau)$ is the complex-valued STFT SDF.

The linear transform selection was based on the next reasons.

The CWT has a good time-frequency resolution, its basis is self-similar, there are many different wavelets allowing to choose the optimal one for each signal analyzed. The argument of the complex AWT SDF has more abilities for the analyzing of the signals with peculiarities than the CWT has. Therefore, AWT is appears to be very useful addition to the CWT. Moreover, there are many different wavelets for AWT, in particular, the Kravchenko-Rvachev analytic wavelets based on the atomic functions [19]. The GT SDF has the best time-frequency localization in the middle of all time-frequency transforms. Sometimes, the AFT comes to the AWT, but in a number of cases it has an independent sense, in particular, when the non-symmetrical window functions have been used.

2.2. NON-LINEAR TRANSFORMS

The Fourier spectrogram [5] is given by

$$P_S f(\omega,\tau) = \left|\dot{S}f(\omega,\tau)\right|^2 = \\ = \left|\int_{-\infty}^{\infty} s(t) g(t-\tau) \exp(-i\omega t) dt\right|^2,$$

where $P_S f(\omega,\tau)$ is the FS SDF.

The Wigner transform [5] is given by

$$P_V f(\omega,\tau) = \int_{-\infty}^{\infty} s\left(\tau + \frac{t}{2}\right) s^*\left(\tau - \frac{t}{2}\right) \exp(-i\omega t) dt,$$

where $P_V f(\omega,\tau)$ is the WiT SDF.

The Choi-Williams transform [8] is given by

$$P_{CW} f(\omega,\tau) = \sqrt{\frac{\gamma}{4\pi}} \int_{-\infty}^{\infty} \frac{\exp(-i\omega t)}{|t|} \times \\ \times \int_{-\infty}^{\infty} \exp\left(-\frac{(u-\tau)^2 \gamma}{4t^2}\right) s\left(u + \frac{t}{2}\right) s^*\left(u - \frac{t}{2}\right) du dt,$$

where $P_{CW} f(\omega,\tau)$ is the ChWT SDF, γ is positive coefficient driving by level of the cross-terms.

The Born-Jordan transform [8] is given by

$$P_{BJ} f(\omega,\tau) = \int_{-\infty}^{\infty} \frac{1}{|t|} \int_{\tau-|t|/2}^{\tau+|t|/2} s\left(q + \frac{t}{2}\right) \times \\ \times s^*\left(q - \frac{t}{2}\right) dq \exp(-i\omega t) dt,$$

where $P_{BJ} f(\omega,\tau)$ is the BJT SDF.

The non-linear transform selection was based on such reasons.

The WiT has good time-frequency resolution which is better than for linear transforms.

The ChWT has the parameter allowing to control by level of the cross-terms appearing in the WiT in case of the multi-component signal analysis.

Another way of the cross-term influence reduction is given by the BJT application.

Being the limit result of the WiT averaging in time- and frequency domains, the FS allows effectively to select the really existent signals and the cross-terms during the WiT interpretation process. In spite of the fact that the FS has quite poor time-frequency resolution, it is appears to be useful for the system spectral analysis.

Moreover, all non-linear transforms are appeared to be useful for the analysis of the signals in case of the non-Gaussian noise presence. The last fact seems to be very important for experts, as long as in many practical cases the Gaussian noise model being traditional for the linear processing methods is appears to be totally inapplicable, and the linear processing is appears to be ineffective.

It is important to note, that instead of each non-linear transform its averaged version can be used. There are, at least, two possible types of such averaging. For example, the ChWT can be replaced by the pseudo Wigner transform (PChWT) given by [21]

$$P_{PCW} f(\tau,\omega;h) = \sqrt{\frac{\sigma}{4\pi}} \int_{-\infty}^{\infty} h(t) \frac{\exp(-i\omega t)}{|t|} \times \\ \times \int_{-\infty}^{\infty} \exp\left(\frac{-(u-\tau)^2 \sigma}{4t^2}\right) s\left(u + \frac{t}{2}\right) s^*\left(u - \frac{t}{2}\right) du dt,$$

Or by the smoothed pseudo Choi-Williams transform (SPChWT) given by

$$P_{SPCW} f(\tau,\omega;h,g) = \sqrt{\frac{\sigma}{4\pi}} \int_{-\infty}^{\infty} h(t) \frac{\exp(-i\omega t)}{|t|} \times \\ \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(v-u) \exp\left(\frac{-(v-\tau)^2 \sigma}{4t^2}\right) \times \\ \times s\left(v + \frac{t}{2}\right) s^*\left(v - \frac{t}{2}\right) dv du dt,$$

where $h(t)$ is the frequency smoothing window in time domain, $g(t)$ is the time smoothing window. The PChWT and the SPChWT allow more effective reduce the cross-terms for the multicomponent signals that the ChWT was allowing. Suddenly, the time-frequency resolution decreases slightly comparing with the ChWT ones.

2.3. OTHER NUMERICAL CHARACTERISTICS

In addition to the SDF of the linear and non-linear transforms listed above, some more characteristics seem to be useful in the system spectral analysis.

The sceleton is defined as the set of the local maxima lines of the SDF module known to the specialists as the 'ridges' too. Some experts believe that in skeleton there is all possible information about signal investigated [5].

The energogram of chosen transform is given by the relation

$$E_P f(T) = \int_{-\infty}^{\infty} |P_l f(T,\tau)|^2 d\tau$$

for the linear transforms and by the relation

$$E_P f(F) = \int_{-\infty}^{\infty} P_n f(F,\tau) d\tau$$

for the non-linear transforms, where $E_P f(T)$ and $E_P f(F)$ are the energograms, $P_l f(T,\tau)$ and $P_n f(F,\tau)$ are the SDF for the linear and non-linear transforms correspondingly, T is the non-dimensional period variable, F is the non-dimensional frequency variable. The distribution of the energy of the analyzed signal or process $s(t)$ along the period or frequency variable is given exactly by the energogram.

The dispersion of module of the SDF coefficients for linear transforms and the mean-square deviation of module of the SDF coefficients for non-linear transforms are given correspondently by the following relations:

$$D_l(T) = \frac{1}{\tau_{\max} - \tau_{\min}} \times$$
$$\times \int_{\tau_{\min}}^{\tau_{\max}} \left[|P_l f(T,\tau)| - \langle |P_l f(T,\tau)| \rangle \right]^2 d\tau,$$

where

$$\langle |P_l f(T,\tau)| \rangle = \frac{1}{(T_{\max} - T_{\min})(\tau_{\max} - \tau_{\min})} \times$$
$$\times \int_{T_{\min}}^{T_{\max}} \int_{\tau_{\min}}^{\tau_{\max}} |P_l f(T,\tau)| \, dT d\tau,$$

$$\sigma_n(\omega) = \left[\frac{1}{\tau_{\max} - \tau_{\min}} \times \right.$$
$$\left. \times \int_{\tau_{\min}}^{\tau_{\max}} \left[P_n f(\omega,\tau) - \langle P_n f(\omega,\tau) \rangle \right]^2 d\tau \right]^{1/2},$$

where

$$\langle P_n f(\omega,\tau) \rangle = \frac{1}{(\omega_{\max} - \omega_{\min})(\tau_{\max} - \tau_{\min})} \times$$
$$\times \int_{\omega_{\min}}^{\omega_{\max}} \int_{\tau_{\min}}^{\tau_{\max}} P_n f(\omega,\tau) d\omega d\tau,$$

$T_{\max}$, $T_{\min}$, $\omega_{\max}$, $\omega_{\min}$, $\tau_{\max}$ and $\tau_{\min}$ are the maximal and minimal values of the T, ω and τ been used for the corresponding SDF calculation.

2.4. DATA REPRESENTATION FORMAT

Thus, for each signal the SDF, the SDF sceleton, the SDF energogram, the dispersion of module of the SDF coefficients for linear transforms and the mean-square deviation of module of the SDF coefficients for non-linear transforms for each used transform were calculated. The results in specially constructed data formats were shown. That formats are useful for the comparing of results given by different transforms.

For the system spectral analysis performing the system of computer mathematics (SCM) MATLAB 7 [22] including packages Wavelet Toolbox, Time-Frequency Toolbox [21], Wave Laboratory [5] and some original software for MATLAB was used.

3. TERAHERTZ UWB ELECTROMAGNETIC PULSE ANALYSYS

The abilities of the new analysis method are demonstrated by the example of investigation of the terahertz electromagnetic pulse, obtained in practice by the nonlinear-optically rectifying of the powerful femtosecond laser pulse in the ZnTe crystal [23]. Maximal generation power has reached up to 3kW per pulse. This terahertz electromagnetic pulse has been used for the measuring of the absorption spectra of the different liquids such as well known solvents, as well as peptide and protein grouts.

The central frequency of the one-dimensional Fourier transform spectral density function of the pulse at the fig. 1, a given has approximately reached to 1 THz ($f_{\min} \approx 0,2$ MHz, $f_{\max} \approx 1,8$ MHz). Its duration in time-domain is equal $\tau_s \approx 1$ ps. This terahertz electromagnetic pulse can be considered as the UWB signal with the fractional bandwidth $\mu \approx 1,6$. The system spectral analysis of the terahertz electromagnetic pulse had been performed and the results obtained have been shown at the fig. 1 and fig. 2.

Fig. 1 deals with ones arrived by the application linear of the transforms. Note, the CWT SDF (fig. 1, b) and AFT SDF (fig. 1, k) were appeared to be most localized at the time-frequency plain. The GT SDF energogram has two local extremums which, apparently, point at the fact that two present signal sidelobes have non-equal time durations and, therefore, have non-equal frequencies. The results obtained with the non-linear transforms usage have been shown at the fig. 2.

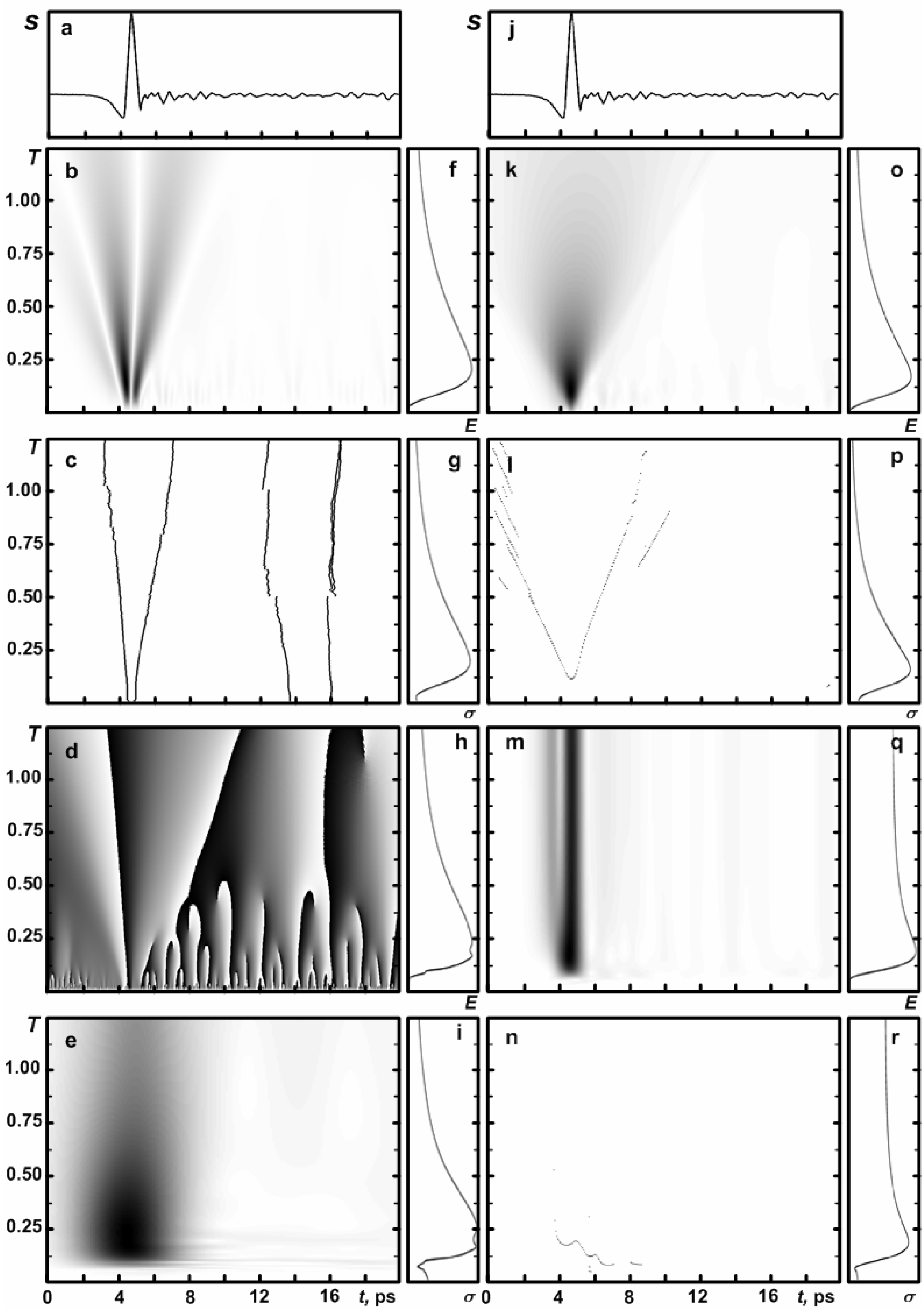

Fig. 1. The results of the terahertz electromagnetic pulse analysis: a, j – signal in time domain, b – CWT SDF with gaus1 wavelet, c – CWT SDF sceleton, d – phase of complex coefficients of AWT with cgau1 wavelet, e – GT SDF module, f – CWT SDF energogram, g – dispersion of CWT SDF coefficients, h – GT SDF energogram, i – dispersion of GT SDF module, k – AFT SDF module, l – AFT SDF sceleton, m – STFT SDF module, n – STFT SDF skeleton, o – AFT SDF energogram, p – dispersion of AFT SDF module, q – STFT SDF energogram, r – dispersion of AFT SDF module.

CONCLUSIONS

- The system spectral analysis as a new integrated method of signal analysis was introduced and applied for the terahertz electromagnetic UWB pulse analysis.
- The main peculiarity of the system spectral analysis is the compensation of disadvantages of ones transforms by the advantages of other transforms.
- The system spectral analysis combines the linear and the non-linear methods of the signal analysis.
- The simultaneous usage of linear and non-linear transforms allows processing the signals at background of as Gaussian as non-Gaussian noises.
- The special data format for the representation of the system spectral analysis of signals was constructed.

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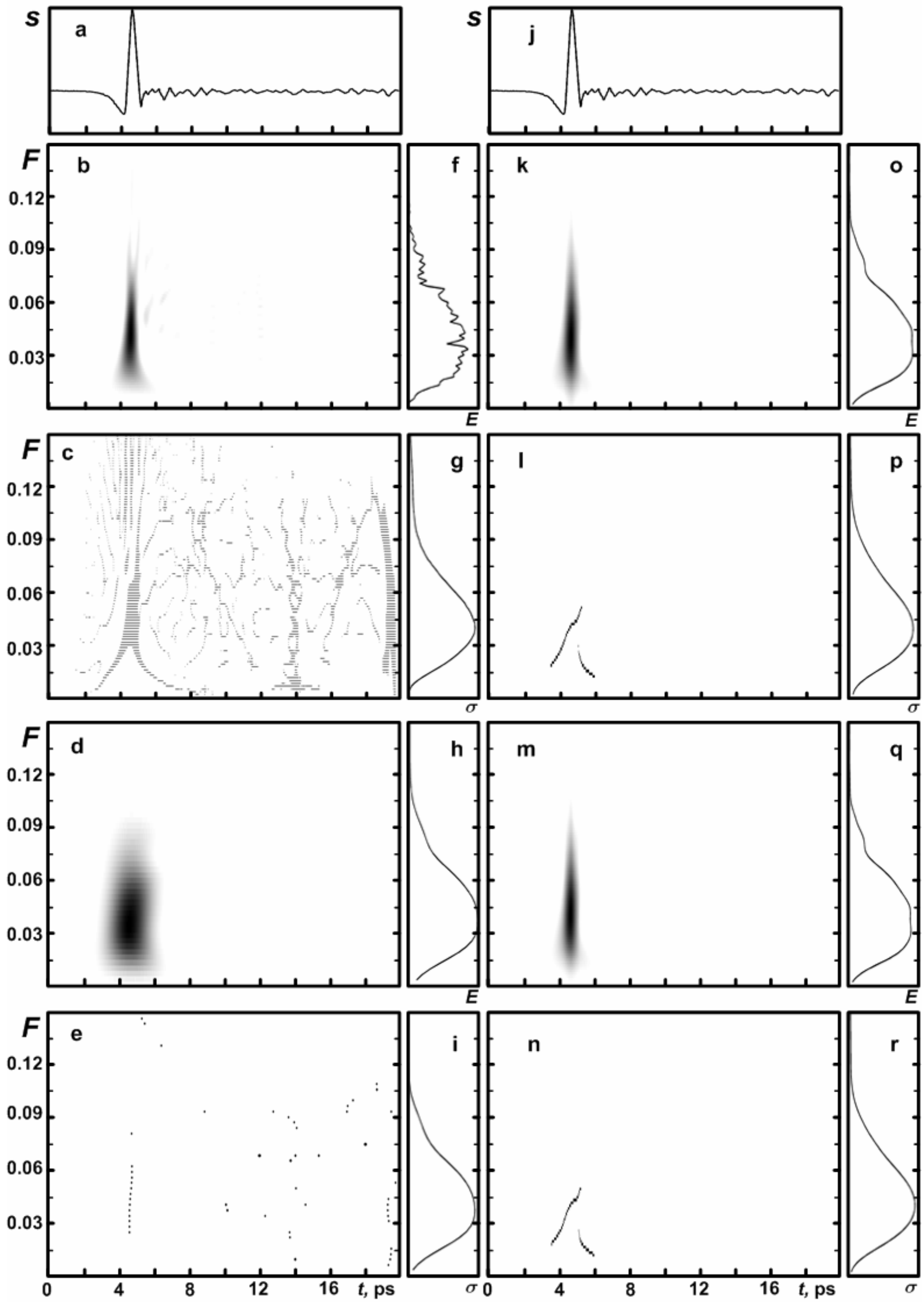

Fig. 2. The results of the terahertz electromagnetic pulse analysis: a, j – signal in time domain, b – PWiT SDF, c – PWiT SDF sceleton, d – FS SDF, e – FS SDF sceleton, f – PWiT SDF energogram, g – dispersion of PWiT SDF coefficients, h – FS SDF energogram, i – dispersion of FS SDF module, k – ChWT SDF module, l – ChWT SDF sceleton, m – BJT SDF module, n – BJT SDF skeleton, o – ChWT SDF energogram, p – dispersion of ChWT SDF module, q – BJT SDF energogram, r – dispersion of BJT SDF module..