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## ELECTRODYNAMICS AND WAVE PROPAGATION

# Excitation of a Conical Slot Antenna

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**Abstract**—A boundary value problem is considered on the scattering of the field of a radial magnetic dipole by a semi-infinite perfectly conducting cone with periodic longitudinal slots. It is shown that the solution of this problem is equivalent to the solution of a singular integral equation with a Cauchy-type kernel. Numerical results are presented for a cone with a single slot. The Fourier coefficients of electromagnetic field components are investigated as functions of the parameters of a cone.

## INTRODUCTION

The design and development of wideband and superwideband radio engineering systems and radar complexes with controlled radiation patterns and polarization are topical problems in modern antenna technique and radar. Of special importance are wideband (superwideband) antennas and reflectors, both directional and omnidirectional. These include, in particular, cones, bicones, as well as conical and planar angular strips (sectors). The development of an adequate mathematical model plays an important role in studying the underlying physical processes and obtaining their important characteristics. However, the solution of an appropriate mathematical problem often requires either the development of new approaches and methods or the modification of the existing ones.

In this paper, we consider a model problem on the excitation of a conical slot antenna by a time-harmonic point source. The conical structure represents a thin semi-infinite perfectly conducting circular cone with longitudinal slots. In [1–3], we investigated the excitation of a perfectly conducting semi-infinite circular cone, with periodic slots cut along the generatrices, by a radial electric dipole. To solve this time-harmonic electromagnetic problem, we used an approach based on the Kontorovich–Lebedev integral representation and the method of the Riemann–Hilbert problem. In certain particular cases, we presented analytic solutions to the problem, which substantially restricted the level and scale of investigations. The application of the method of the Riemann–Hilbert problem to the scattering of electromagnetic waves by impedance structures is associated with great difficulties; therefore, it was suggested to reduce the problem to solving a singular integral equation with the use of the method of discrete singularities [4].

The aim of this work is to construct an algorithm for reducing the problem of exciting a perfectly conducting semi-infinite circular cone with periodic longitudinal slots by a radial magnetic dipole to a singular integral equation and carry out a numerical experiment.

## 1. FORMULATION OF THE PROBLEM. FUNCTIONAL EQUATIONS

Let us consider a semi-infinite perfectly conducting circular cone with $N$ periodic slots cut along the generatrices. We denote by $2\gamma$ the opening angle of the cone, by $l = 2\pi/N$, the period of the structure, and by $d$, the width of a slot ($l$ and $d$ are bihedral angles formed by the planes passing through the axis of the cone and the edges of two neighboring conical strips). We introduce a spherical system of coordinates $r$, $\theta$, $\varphi$ with the origin at the vertex of the cone (Fig. 1). In this system, the cone is represented by the set of points

$$\sum = \{(r, \theta, \varphi) \in R^3 : r \in [0, +\infty), \theta = \gamma, \varphi \in L\},$$

where

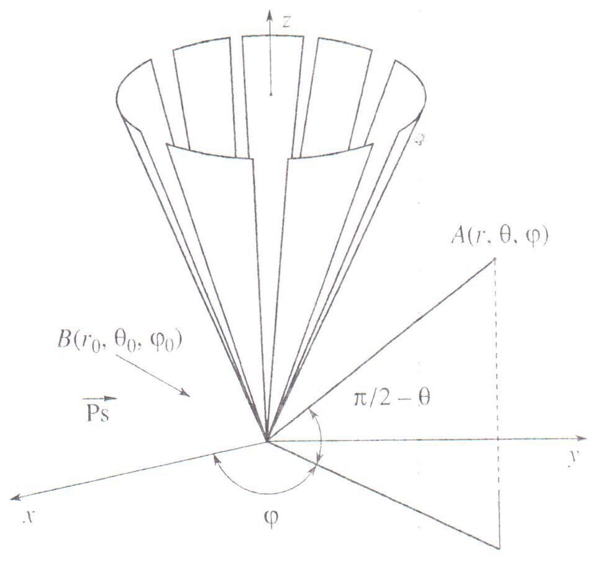


**Fig. 1.** Geometry of the problem.

$$L = \bigcup_{s=1}^{N} L_s, \quad L_s = ((s-1)l + d/2, sl - d/2),$$

$$\text{and } CL = [0.2\pi]\backslash L;$$

$L_s$ corresponds to a conical strip with number $s$.

Let us place a radial magnetic dipole with a unit moment at point $B(r_0, \theta_0, \varphi_0)$; the time variation of the field radiated by this dipole is given by $\exp(i\omega t)$. The total field $\vec{E}$, $\vec{H}$, which we represent as a sum of the field ($\vec{E}_0$, $\vec{H}_0$) of the dipole and the field ($\vec{E}_s$, $\vec{H}_s$) scattered by the cone, satisfies the Maxwell equations, the boundary condition (the vanishing of the tangential components of the electric field on the strips of the cone), the radiation condition at infinity, and a condition that the energy must be finite. The problem in this statement has a unique solution. To solve this electromagnetic boundary value problem, it is convenient to introduce a Debye magnetic potential in terms of which the components of the electromagnetic field are expressed. Thus, it is required to find Debye potential $u$ that satisfies a homogeneous Helmholtz equation outside the cone and the source, the Neumann boundary condition on the strips of the cone, the limit absorption principle, and the edge condition near the irregularities of the boundary (near the edges of the strips and the vertex of the cone). We represent the required potential as $u = u_0 + u_s$, where $u_0 = \frac{1}{r_0}\frac{\exp(-ikR)}{R}$ is the Debye potential for the field of the source, $k$ is a wavenumber, $\operatorname{Im} k \le 0$, $R = |\vec{r} - \vec{r}_0|$, and $u_s$ is the potential for the scattered field. To solve the second boundary value problem of mathematical physics, we apply the Kontorovich–Lebedev integral transform

$$\tilde{g}(\tau) = \int_0^{+\infty} g(r)\frac{H_{i\tau}^{(2)}(kr)}{\sqrt{r}}dr, \tag{1}$$

$$g(r) = -\frac{1}{2}\int_0^{+\infty} \tau\sinh\pi\tau\exp(\pi\tau)\tilde{g}(\tau)\frac{H_{i\tau}^{(2)}(kr)}{\sqrt{r}}d\tau, \tag{2}$$

where $H_{i\tau}^{(2)}(kr)$ is the Hankel function of the second kind. The unknown function $u_s$ is sought for in the form of the Kontorovich integral (1), (2):

$$u_s = -\frac{1}{2}\int_0^{+\infty} \tau\sinh\pi\tau\exp(\pi\tau)\frac{H_{i\tau}^{(2)}(kr)}{\sqrt{r}} \times \sum_{m=-\infty}^{\infty} a_m(\tau, k)V_{m\tau}(\theta, \varphi)d\tau, \tag{3}$$

$$a_{m\tau} = \frac{(-1)^{m+1}}{r_0}\frac{\pi}{\cosh\pi\tau}\exp(-im\varphi_0)\frac{\Gamma\left(\frac{1}{2} - m + i\tau\right)}{\Gamma\left(\frac{1}{2} + m + i\tau\right)}$$

$$\times \frac{H_{i\tau}^{(2)}(kr_0)}{\sqrt{r_0}}P_{-1/2+i\tau}^{m}(-\cos\theta_0)\frac{d}{d\gamma}P_{-1/2+i\tau}^{m}(\cos\gamma),$$

$$V_{m\tau} = \sum_{n=-\infty}^{+\infty} x_{m,n}(\tau)\frac{P_{-1/2+i\tau}^{m+nN}(\pm\cos\theta)}{\frac{d}{d\gamma}P_{-1/2+i\tau}^{m+nN}(\pm\cos\gamma)} \tag{4}$$

$$\times \exp(i(m+nN)\varphi), \quad \gamma < \theta_0,$$

where $\Gamma(z)$ is the gamma function, $P_\mu^m(\cos\theta)$ is the associated Legendre function of the first kind, $x_{m,n}$ are unknown coefficients, $\nu = \frac{m}{N} - m_0$, $-1/2 \le \nu < 1/2$, and $m_0$ is an integer closest to $m/N$. The upper and lower signs in (3) and (4) correspond to the domains $0 < \theta < \gamma$ and $\gamma < \theta < \pi$, respectively. To obtain functional equations containing $x_{m,n}$, we use the boundary conditions on the strips of the cone and the matching condition in the slots:

$$\frac{\partial u}{\partial \theta} = 0, \quad \theta = \gamma, \quad \varphi \in L, \tag{5}$$

$$u^+ = u^-, \quad \theta = \gamma, \quad \varphi \in CL, \tag{6}$$

where $u^\pm = u|_{\theta = \gamma \pm 0}$.

The application of conditions (5) and (6) (due to the periodicity of the structure, we consider them on a period) results in the following system of functional relations:

$$\sum_{n=-\infty}^{+\infty} x_{m,n}\exp(inN\varphi) = \exp(im_0N\varphi), \tag{7}$$

$$\varphi \in L_0\colon \frac{\pi d}{l} < |N\varphi| \le \pi,$$

$$\sum_{n=-\infty}^{+\infty} \frac{1}{N(n+\nu)}\frac{|n|}{n}(1-\varepsilon_n)x_{m,n}\exp(inN\varphi) = 0, \tag{8}$$

$$\varphi \in CL_0\colon |N\varphi| \le \frac{\pi d}{l},$$

$$\frac{1}{N(n+\nu)}\frac{|n|}{n}(1-\varepsilon_n) = \frac{(-1)^{(n+\nu)N+1}\cosh\pi\tau}{\pi(\sin\gamma)^2}\frac{\Gamma(1/2 + i\tau + (n+\nu)N)}{\Gamma(1/2 + i\tau - (n+\nu)N)} \tag{9}$$

$$\times \frac{1}{\frac{d}{d\gamma}P^{(n+\nu)N}_{-1/2+i\tau}(\cos\gamma)\frac{d}{d\gamma}P^{(n+\nu)N}_{-1/2+i\tau}(-\cos\gamma)}.$$

When $N(n+\nu) \gg 1$, the following estimate holds for $\varepsilon_n$ (9):

$$\varepsilon_n = O\left(\frac{1}{N^2(n+\nu)^2}\right).$$

Below, we consider the functional relations (7) and (8) as equations in the unknown coefficients $x_{m,n}$ that are contained in the Hilbert space of sequences $\{\xi_p\}$,

$$\sum_{p=-\infty}^{+\infty} |\xi_p|^2(1+|p|)^{-1} < +\infty.$$

## 2. SINGULAR INTEGRAL EQUATION

Let us introduce

$$\psi = -\frac{|\varphi|}{\varphi}\pi + N\varphi, \quad \eta_{m,n} = x_{m,n}(-1)^{n-m_0},$$

$$\delta = \frac{l-d}{l}\pi.$$

Then, system (7), (8) is reduced to

$$\sum_{n=-\infty}^{\infty} \eta_{m,n}\exp(in\psi) = \exp(im_0\psi), \quad \psi \in L_0\colon |\psi| < \delta$$

$$\sum_{n=-\infty}^{\infty} \frac{1}{N(n+\nu)}\frac{|n|}{n}(1-\varepsilon_n)\eta_{m,n}\exp(in\psi) = 0, \quad (10)$$

$$\psi \in CL_0\colon \delta < |\psi| \le \pi$$

Multiplying both sides of (10) by $\exp(i\nu\psi)$ and differentiating with respect to $\psi$, we obtain

$$\sum_{n=-\infty}^{\infty} \frac{|n|}{n}(1-\varepsilon_{m,n})\eta_{m,n}\exp(in\psi) = 0, \quad \psi \in CL_0.$$

Since some coefficients are lost during the differentiation, we add an additional condition

$$\sum_{n=-\infty}^{\infty} \frac{1}{N(n+\nu)}\frac{|n|}{n}(1-\varepsilon_{m,n})(-1)^n\eta_{m,n} = 0.$$

Thus, the original electromagnetic problem reduces to the following system of equations in $\eta_{m,n}$:

$$\sum_{n=-\infty}^{\infty} \eta_{m,n}\exp(in\psi) = \exp(im_0\psi), \quad \psi \in L_0, \quad (11)$$

$$\sum_{n=-\infty}^{\infty} \frac{|n|}{n}(1-\varepsilon_{m,n})\eta_{m,n}\exp(in\psi) = 0, \quad \psi \in CL_0, (12)$$

subject to the additional condition

$$\sum_{n=-\infty}^{\infty} \frac{1}{N(n+\nu)}\frac{|n|}{n}(1-\varepsilon_{m,n})(-1)^n\eta_{m,n} = 0. \quad (13)$$

Let us introduce the function

$$F(\psi) = \sum_{n=-\infty}^{\infty} \frac{|n|}{n}(1-\varepsilon_{m,n})\eta_{m,n}\exp(in\psi), \quad (14)$$

$$\psi \in [-\pi, \pi]$$

It follows from (12) that

$$F(\psi) = 0, \quad \psi \in CL_0. \quad (15)$$

Taking into account (13)–(15), we express the coefficients $\eta_{m,n}$ in terms of $F(\psi)$:

$$\eta_{m,n} = \frac{1}{2\pi}\frac{|n|}{n}(1-\varepsilon_{m,n})\int_{L_0} F(\xi)\exp(-in\xi)d\xi,$$

$$n \ne 0, \quad (16)$$

$$\eta_{m,0} = -\frac{1}{A^{\nu}_{m,\tau}}\frac{1}{2N\pi}\int_{L_0} F(\xi)\left(\frac{\pi\exp(i\nu\xi)}{\sin\pi\nu} - \frac{1}{\nu}\right)d\xi,$$

where

$$A^{\nu}_{m\tau} \equiv \frac{1}{N(n+\nu)}\frac{|n|}{n}(1-\varepsilon_{m,n})\big|_{n=0}, \quad \frac{1}{1-\varepsilon_{m,n}} \equiv 1-\tilde{\varepsilon}_{m,n}.$$

Taking into account that

$$\sum_{n\ne 0} \frac{(-1)^n}{n+\nu}\exp(-in\xi) = \frac{\pi\exp(i\nu\xi)}{\sin\pi\nu} - \frac{1}{\nu},$$

$$\sum_{n\ne 0} \frac{|n|}{n}\exp(in(\psi-\xi)) = -i\cot\frac{\xi-\psi}{2},$$

from (11), (13), and (16) we obtain the following singular integral equation (SIE) with a Cauchy kernel for the function $F(\xi)$:

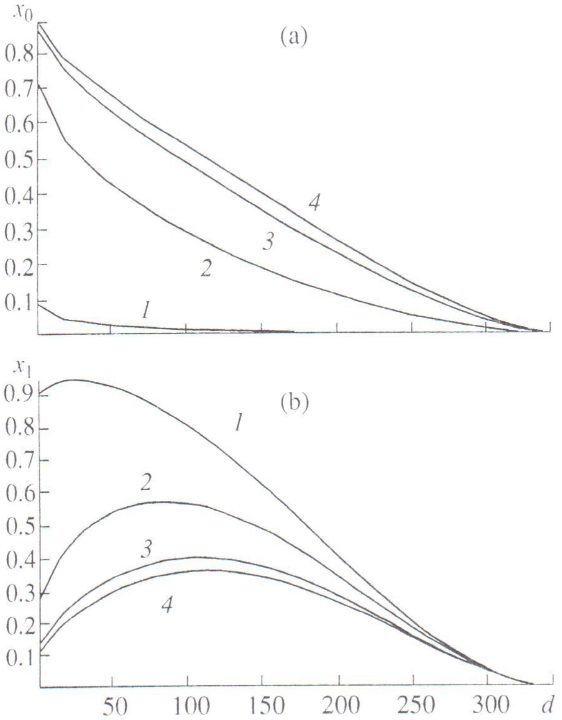


**Fig. 2.** The dependence of (a) $|x_0|$ and (b) $|x_1|$ on the slot width $d$ for various values of $\gamma$; curves *1–4* correspond to $\gamma = 5°, 30°, 60°$, and $90°$, respectively.

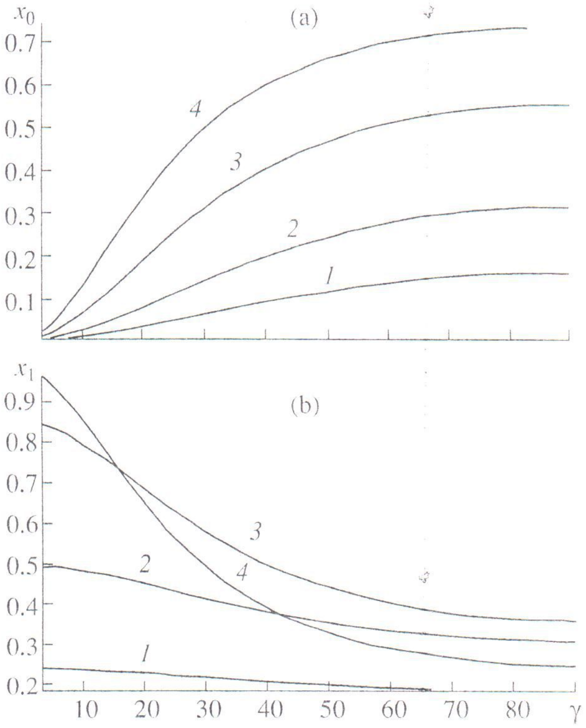


**Fig. 3.** The dependence of (a) $|x_0|$ and (b) $|x_1|$ on the half-flare angle $\gamma$ for various values of $d$; curves *1–4* correspond to $d = 243°, 180°, 90°$, and $30°$, respectively.

where

$$F(t) = \frac{V(t)}{\sqrt{1-t^2}}, \tag{22}$$

$t_p^q = \cos\frac{2p-1}{2q}\pi$ are the roots of the Chebyshev polynomial of the first kind, and $t_{oj}^q = \cos\frac{j}{q}\pi$ are the roots of the Chebyshev polynomial of the second kind.

Taking into account relation (22) between $F(t)$ and $V(t)$ and applying the Gauss quadrature formula

$$\int_{-1}^{1}\frac{g(t)}{\sqrt{1-t^2}}dt = \sum_{p=1}^{q} g(t_p^q)\frac{\pi}{q},$$

we obtain the following formulas for calculating coefficients $\eta_0$ and $\eta_n$:

$$\eta_0 = -\frac{1}{A_\tau}\frac{i\delta^2}{2}\sum_{p=1}^{q} V(t_p^q)t_p^q\frac{1}{q} \tag{23}$$

$$\eta_n = \frac{1}{2}\frac{|n|}{n}(1-\varepsilon_n)\delta\sum_{i=1}^{k} V(t_p^q)\exp(-in\delta t_p^q)\frac{1}{q}. \tag{24}$$

Solving SLAE (20), (21) yields $V(t_p^q)$, using which we determine $\eta_0$ and $\eta_n$ from Eqs. (23), (24), which are related to the sought-for coefficients $x_n(d, \gamma, \tau)$. We investigate $x_n$ as functions of the problem parameters. The table presents coefficients $x_n$ versus the slot width in the case of a cone with a single slot ($N = 1$, $\tau = 1$, $\gamma = \pi/8$). The imaginary parts of the coefficients are negligible in absolute value as compared with the real parts; in the case of a narrow conical strip, $|x_0| < |x_n|$, $n = 1, 2, 3, \ldots, 19$. As the strip width decreases, the signs of real parts of the coefficients with different numbers alternate, while their moduli decrease. In the limit case of a narrow conical strip ($\delta = 1°$, $d = 359$), the moduli are of order $10^{-4}$. This result is in good agreement with the asymptotics for the coefficients in the case of a narrow strip ($\delta \ll 1$) [5]:

$$x_0 \approx \frac{\sin^2(\pi\delta/2)}{A_\tau + \sin^2(\pi\delta/2)}, \quad x_n \approx \frac{(-1)^n}{1-\varepsilon_n}n\frac{A_\tau \sin^2(\pi\delta/2)}{A_\tau + \sin^2(\pi\delta/2)}.$$

Figure 2 presents $|x_0|$ versus the slot width for various $\gamma$ ($N = 1$ and $\tau = 1$). As the slot width increases (the width of a conical strip decreases), $|x_0(d)|$ monotonically decreases to zero; the smaller the half-flare angle $\gamma$ of the cone, the gentler the curves.

In contrast to $|x_0(d)|$, the graph of $|x_1(d)|$ has a maximum that moves as $\gamma$ is varied (Fig. 2b; $N = 1$ and $\tau = 1$). The graphs of $|x_0|$ and $|x_1|$ versus $\gamma$ for various $d$ are shown in Figs. 3a and 3b.

$$\frac{1}{\pi}\int_L \frac{F(\xi)}{\xi-\psi}d\xi + \frac{1}{\pi}\int_L K(\xi,\psi)F(\xi)d\xi = i\exp(im_0\psi), \tag{17}$$

$$\psi \in L_0$$

and the additional condition

$$\int_{CL} F(\xi)d\xi = 0,$$

where

$$K(\xi,\psi) = \frac{1}{2}\cot\frac{\xi-\psi}{2} - \frac{1}{\xi-\psi} - \frac{i}{2N}\left(\frac{\pi\exp(i\nu\xi)}{\sin\pi\nu} - \frac{1}{\nu}\right)\frac{1}{A^{\nu}_{m\tau}} - \frac{i}{2}\sum_{n\neq 0}\frac{|n|}{n}\tilde{\varepsilon}_{m,n}\exp[in(\psi-\xi)].$$

By the change of variables $\psi/\delta = t_0$ and $\xi/\delta = t$, we reduce the integrals in (17) to the integrals over the interval $(-1, 1)$:

$$\begin{cases} \frac{1}{\pi}\int_{-1}^{1}\frac{F(t)}{t-t_0}dt + \frac{\delta}{\pi}\int_{-1}^{1}K(t,t_0)F(t)dt = i\exp(im_0\delta t_0), \\ |t_0| < 1, \\ \frac{1}{\pi}\int_{-1}^{1}F(t)dt = 0, \end{cases} \tag{18}$$

where

$$K(t,t_0) = \frac{1}{2}\cot\frac{t-t_0}{2}\delta - \frac{1}{(t-t_0)\delta} - \frac{i}{2N}\left(\frac{\pi\exp(i\nu\delta t)}{\sin\pi\nu} - \frac{1}{\nu}\right)\frac{1}{A^{\nu}_{m\tau}} - \frac{i}{2}\sum_{n\neq 0}\frac{|n|}{n}\tilde{\varepsilon}_{m,n}\exp(-in(t-t_0)\delta).$$

Let us consider the case of axially symmetric excitation ($\theta_0 = \pi$, $\varphi_0 = 0$, $m = 0$; henceforth, we omit index $m$) of a cone with a single slot ($N = 1$, $\nu = 0$). Taking into account that

$$\lim_{\nu\to 0}\left(\frac{\pi}{\sin\pi\nu} - \frac{1}{\nu}\exp(-i\nu\delta t)\right) = it\delta,$$

we transform SIE (18) into

$$\begin{cases} \frac{1}{\pi i}\int_{-1}^{1}\frac{F(t)}{t-t_0}dt + \frac{\delta}{\pi i}\int_{-1}^{1}K(t,t_0)F(t)dt = 1, \quad |t_0| < 1 \\ \int_{-1}^{1}F(t)dt = 0 \end{cases}$$

$$K(t,t_0) = \frac{1}{2}\cot\frac{t-t_0}{2}\delta - \frac{1}{(t-t_0)\delta} + \frac{1}{2}t\delta\frac{1}{A_\tau} - \sum_{n=1}^{+\infty}\tilde{\varepsilon}_n\sin\delta n(t-t_0). \tag{19}$$

In this case,

$$\eta_0 = -\frac{i}{A_\tau}\frac{\delta^2}{2\pi}\int_{-1}^{1}F(t)dt,$$

$$\eta_n = \frac{1}{2\pi}\frac{|n|}{n}(1-\varepsilon_n)\delta\int_{-1}^{1}F(t)e\exp(-in\delta t)dt,$$

$$A_\tau = \frac{\cosh\pi\tau}{\pi\sin^2\gamma}\frac{1}{P^1_{-1/2+i\tau}(\cos\gamma)P^1_{-1/2+i\tau}(-\cos\gamma)}.$$

## 3. ANALYSIS OF NUMERICAL RESULTS

To solve SIE (19), we apply the method of discrete singularities [4]. According to this method, SIE (19) is equivalent to the following system of linear algebraic equations (SLAE):

$$\sum_{p=1}^{q}\frac{V_k(t_i^k)}{t_p^q - t_{oj}^q}\frac{1}{q} + \delta\sum_{i=1}^{k}K(t_p^q, t_{oj}^q)V_q(t_p^q)\frac{1}{q} = i, \quad j = \overline{1, q-1}, \tag{20}$$

$$\sum_{i=p}^{q}V_q(t_p^k) = 0, \quad j = q, \tag{21}$$

Coefficients $x_n$ versus the slot width for a cone with a single slot ($N = 1$, $\tau = 1$, and $\gamma = \pi/8$)

| $x_n \backslash d$ | 15° | 35° | 350° | 359° |
|---|---|---|---|---|
| $x_0$ | $0.461534, \; -i9.32635 \times 10^{-21}$ | $0.368127, \; -i1.0545 \times 10^{-20}$ | $0.0003122, \; -i2.17034 \times 10^{-23}$ | $3.12614 \times 10^{-6}, \; -i4.75435 \times 10^{-28}$ |
| $x_1$ | $-0.536123, \; i9.75586 \times 10^{-17}$ | $-0.617233, \; i1.05795 \times 10^{-16}$ | $-0.00210983, \; i9.33418 \times 10^{-21}$ | $-2.11449 \times 10^{-5}, \; i1.01913 \times 10^{-21}$ |
| $x_2$ | $-0.5293, \; i4.71403 \times 10^{-17}$ | $-0.57456, \; i7.2636 \times 10^{-17}$ | $0.0038871, \; -i3.0774 \times 10^{-20}$ | $3.90671 \times 10^{-5}, \; -i1.00477 \times 10^{-21}$ |
| $x_3$ | $-0.517882, \; i1.51123 \times 10^{-17}$ | $-0.50675, \; i2.30636 \times 10^{-17}$ | $-0.0057173, \; i5.63574 \times 10^{-20}$ | $-5.77338 \times 10^{-5}, \; i9.20313 \times 10^{-22}$ |
| $x_4$ | $-0.502147, \; -i6.60919 \times 10^{-17}$ | $-0.418507, \; -i8.67238 \times 10^{-17}$ | $0.0075352, \; -i1.2775 \times 10^{-19}$ | $7.65965 \times 10^{-5}, \; -i1.18607 \times 10^{-21}$ |
| $x_5$ | $-0.482329, \; i1.88262 \times 10^{-16}$ | $-0.315695, \; i9.2823 \times 10^{-17}$ | $-0.00931763, \; i2.11169 \times 10^{-19}$ | $-9.55276 \times 10^{-5}, \; i1.1063 \times 10^{-21}$ |
| $x_6$ | $-0.458577, \; i4.16875 \times 10^{-16}$ | $-0.205323, \; i2.21983 \times 10^{-16}$ | $0.0110506, \; -i2.28154 \times 10^{-19}$ | $0.00011448, \; -i1.20046 \times 10^{-21}$ |
| $x_7$ | $-0.431314, \; i2.57818 \times 10^{-16}$ | $-0.094685, \; -i3.65922 \times 10^{-16}$ | $-0.0127228, \; i2.93294 \times 10^{-19}$ | $-0.0001335, \; i1.21391 \times 10^{-21}$ |
| $x_8$ | $-0.400831, \; -i5.74405 \times 10^{-16}$ | $0.009066, \; -i8.83195 \times 10^{-16}$ | $0.014324, \; -i4.99664 \times 10^{-19}$ | $0.00015243, \; -i1.50707 \times 10^{-21}$ |
| $x_9$ | $-0.36744, \; -i1.36244 \times 10^{-15}$ | $0.099583, \; -i2.65287 \times 10^{-16}$ | $-0.0158449, \; i5.34165 \times 10^{-19}$ | $-0.00017141. \; i1.84595 \times 10^{-21}$ |
| $x_{10}$ | $-0.331683, \; -i3.62072 \times 10^{-16}$ | $0.171657. \; i1.08953 \times 10^{-15}$ | $0.0172767, \; -i8.01621 \times 10^{-19}$ | $0.00019038, \; -i1.54399 \times 10^{-21}$ |
| $x_{11}$ | $-0.293933, \; i8.34459 \times 10^{-16}$ | $0.221596, \; i1.38152 \times 10^{-15}$ | $-0.0186114, \; i9.61951 \times 10^{-19}$ | $-0.00020934, \; i1.8665 \times 10^{-21}$ |
| $x_{12}$ | $-0.25459, \; i1.93611 \times 10^{-15}$ | $0.247459, \; i5.67961 \times 10^{-17}$ | $0.0198417, \; -i8.57281 \times 10^{-19}$ | $0.00022829, \; -i1.97214 \times 10^{-21}$ |
| $x_{13}$ | $-0.214236, \; i1.73773 \times 10^{-15}$ | $0.249135, \; -i1.13747 \times 10^{-15}$ | $-0.0209609, \; i1.37938 \times 10^{-18}$ | $-0.00024724, \; i2.0511 \times 10^{-21}$ |
| $x_{14}$ | $-0.173358, \; i1.50512 \times 10^{-15}$ | $0.228354, \; -i1.49144 \times 10^{-15}$ | $0.0219629. \; -i1.08405 \times 10^{-18}$ | $0.00026617, \; -i1.86691 \times 10^{-21}$ |
| $x_{15}$ | $-0.132479, \; i3.95972 \times 10^{-16}$ | $0.188556, \; -i1.43488 \times 10^{-15}$ | $-0.0228427, \; i1.48376 \times 10^{-18}$ | $-0.00028508, \; i2.77874 \times 10^{-21}$ |
| $x_{16}$ | $-0.092039, \; -i7.11995 \times 10^{-16}$ | $0.134444, \; -i6.31763 \times 10^{-16}$ | $0.0235957, \; -i1.73584 \times 10^{-18}$ | $0.00030398, \; -i2.53744 \times 10^{-21}$ |
| $x_{17}$ | $-0.052533. \; -i8.01572 \times 10^{-16}$ | $0.0715083. \; i1.11041 \times 10^{-16}$ | $-0.0242182, \; i2.07546 \times 10^{-18}$ | $-0.00032287, \; i3.53216 \times 10^{-21}$ |
| $x_{18}$ | $-0.014465, \; -i1.40176 \times 10^{-15}$ | $0.0057734. \; i1.29206 \times 10^{-15}$ | $0.0247076. \; -i1.84056 \times 10^{-18}$ | $0.00034173, \; -i3.75449 \times 10^{-21}$ |
| $x_{19}$ | $0.021816, \; -i1.87698 \times 10^{-15}$ | $-0.056986. \; i1.42086 \times 10^{-15}$ | $-0.0250617. \; i2.19631 \times 10^{-18}$ | $-0.00036058, \; i3.74348 \times 10^{-21}$ |

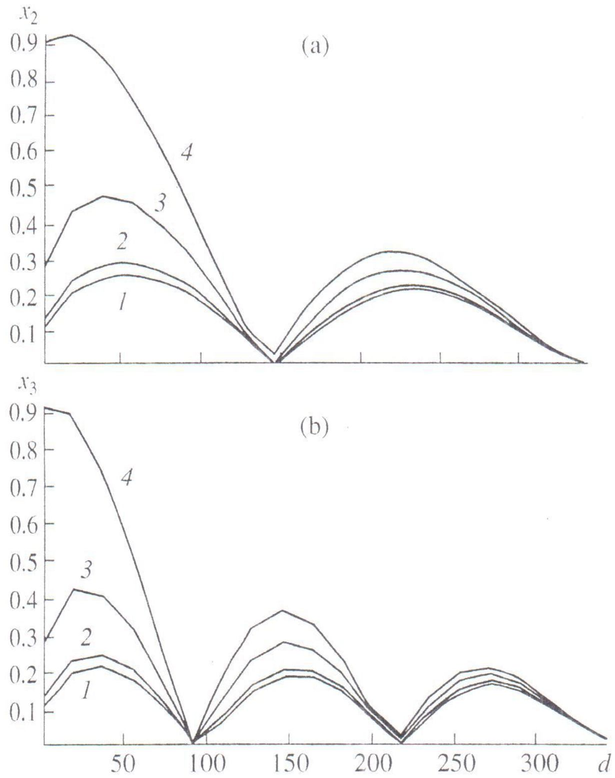


**Fig. 4.** The dependence of (a) $|x_2|$ and (b) $|x_3|$ on the slot width $d$ for various values of $\gamma$; curves *1–4* correspond to $\gamma$ = 90°, 60°, 30°, and 5°, respectively; $N = 1$ and $\tau = 1$.

The behavior of $|x_2(d)|$ and $|x_3(d)|$ for various $\gamma$ is illustrated in Figs. 4a and 4b. As the number of coefficients increases, the number of maxima and minima of the curves also increases, while their magnitudes depend on $\gamma$. Comparing the behavior of $|x_1(d)|$, $|x_2(d)|$, and $|x_3(d)|$ (Figs. 2b, 4a, and 4b), we observe that, as the number of $x_n$ increases, the maxima move and their magnitudes decrease for appropriate fixed values of $\gamma$.

## CONCLUSION

In this work, we proposed an algorithm for solving a model problem for the excitation of a conical slot antenna. This algorithm consists in reducing the problem to a singular integral equation with a Cauchy kernel. The numerical solution of the singular equation is performed by the method of discrete singularities for a cone with a single slot. On the basis of this solution, we investigated the Fourier coefficients of the electromagnetic field components as functions of the angular dimensions of the conical structure. For fixed slot width, the absolute values of the coefficients of nonzero harmonics increase as the opening angle of the cone increases. In the limit case of a single narrow strip, the numerical results are in good agreement with analytical results.

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