UDC 621.391

MODELS OF STRUCTURAL COHERENCE IN TELECOMMUNICATION SYSTEMS

V.V. POPOVSKY
Kharkiv National University of Radioelectronics

Abstract *- Mathematical models of structural connectivity for fixed and dynamic systems are considered. Ezary-Proshan and Polessky estimations are given for fixed systems while for dynamic systems these estimations are determined in the state space. Quality of state estimation under different connectivity of vector components of dynamic system nodes is analyzed. It is shown that Tchebotarev-Achaev basic model approaches the procedure of stochastic synchronization of states. The presence of connectivity in the network structures, such as infocommunication systems, provides the acquisition of properties for the reliability and survivability of networks. For fixed (invariable with time parameters) networks connection is displayed as connectivity matrix, which may have a binary structure or consist of quantitative data de-fining the level of the connection (probability, number of channels, distance, etc.). The co-efficient of connectivity or connectivity probability for such networks is found by the approximate methods of Ezary-Proschan, Polesski etc.*

For dynamic (with time-varying elements of connection) network connectivity values can be obtained through the current assessment of the state of connectivity for each node from all adjacent nodes. For real posteriori evaluation of connectivity it is appropriate to use the state-space model that allows us to characterize the dynamic systems as deterministic and stochastic. Practice has shown that the state of the process allows to use the methods of stochastic approximation under the condition of choosing the sampling rate. The analysis of multi-dimensional differential systems shows that the existence of connections between components of the system cannot have arbitrary values. A large number of connections leads to unstable regimes. In the other extreme case: under the absence of reciprocal connection the system loses its system properties (integrity, emergence). The connection between random processes improves accuracy (reduced values of a posterior variance) of the estimates of the components for these processes compared to that case when data of the process are independent.

Анотація *– Запропоновано математичні моделі зв'язності структур для фіксованих і динамічних систем. Дано оцінки Езар-Прошана і Поліського для стаціонарних систем, у той час як для динамічних систем ці оцінки визначаються в просторі станів. Проаналізовано якість оцінки стану при різній зв'язності компонент вектору динамічних вузлів системи. Показано, що базова модель Чеботарьова-Агаєва апроксимує процедуру стохастичної синхронізації станів.*

Аннотация *– Предложены математические модели связности структур для фиксированных и динамических систем. Даны оценки Эзари-Прошана и Полесского для стационарных систем, в то время как для динамических систем эти оценки определяются в пространстве состояний. Проанализировано качество оценки состояния при различной связности компонент вектора динамических узлов системы. Показано, что базовая модель Чеботарева-Агаева аппроксимирует процедуру стохастической синхронизации состояний.*

Introduction

Mathematical model of system coherence $S(x_n)$ is the connection matrix $A=\{a_{ij}\}$, which determines the lengths of the paths between i and j vertices (nodes) [1]. Connectivity of network is often associated with their reliability, because the assumption that an increase in the number of connections (edges) increases the reliability of the transmission of information between any two nodes is rather logical.

The model determines the reliability as a connectivity matrix $A=\{a_{ij}\}$ typically used in the design phase or network upgrades. Therefore, this model should be considered a priori characteristic unlike a posteriori that is defined as the percentage of time during which the requirements of performance criteria are met.

The simplest model of connectivity $A=\{a_{ij}\}$ is a binary matrix $a_{ij}\in[0,1]$, which gives an idea of presence or absence of communication between i and j nodes. A matrix, whose elements a_{ij}, $i,j=1,2,...n$, $i\neq j$ numerically parameterize various properties of links: the length of the lines, the probability of communication, channel capacity etc. [3, 4], has greater content-richness. With the probabilistic characterization the elements a_{ij} are random values and known methods can calculate the probability of connectivity P_{ij} for all nodes, especially if their number is in units of one. With the increase of n the computational complexity increases proportionally $n!$. For the numerical results at big values of n we use various approximate estimates such as estimates of Ezary - Proshan [2] , Poleski [3, 4], etc. Calculated estimates characterize the state of the connection probability for a fixed (time-invariant) random system $S(x_n)$.

The connection matrix $A(t)$ of the dynamical system $S(x_n(t))$, whose elements $a_{ij}(t)$ represent random processes, has greater generality. In this view, the connection matrix can display not only structural but also the functional properties of the simulated system $S(x_n(t))$ [5, 6].

I. Description of the models for dynamic structures

Due to the development of cybernetics, control theory and information communications, dynamical models of systems in the space of states have acquired a strong theoretical foundation and wide practical application. Thus, within the changing conditions, the most constructive methods are those based on systems of differential or difference equations, where the state ith node (load, queue length, delay, jitter, etc.) is considered as $x_i(t)$ variable. A model of the structure of the network section, adjacent to the ith node, determines the appropriate coefficients $\alpha_{ij}(t)$ before the interaction functions of this node with all adjacent j nodes.

The general linear equation for the dynamics model of stochastic controlled system $S(x(t),u(t),t)$ has the following form [8]:

$$dx_i(t)/d(t)=A(t)x_i(t)+B(t)u_i(t)+G(t)\xi_i(t),\ i=\overline{1,n}, \tag{1}$$

where $\xi_i(t)$ is the virtual vector of generating white Gaussian noise. The dimension of the matrix corresponds to the dimension of the state vector $x_i(t)$.

In order to provide observability, the system of equations (1) should be considered together with the system of observation equations (measurement)

$$y_i(t)=H(t)x_i(t)+v_i(t), \tag{2}$$

where $v_i(t)$ is the noise in the observation channels from those adjacent with the i-th nodes non-correlated with $\xi_i(t)$. The matrix $H(t)$ determines the scale of the state vectors $x_i(t)$ and in particular it can be $H(t)=I$.

Clearly, in those particular cases when $\xi(t)=v(t)=0$ the $S(x)$ system is deterministic and then simplification is allowed, facilitating the analysis and synthesis of systems. In the general case, the representations of the models (1), (2) are rather universal for linear dynamic systems.

For the complete system it is unacceptable to assume that in equation (1) all the elements of connectivity matrix are $a_{ij}(t)=0$ under $i \neq j$. Besides, the equations (1) become independent, and the system $S(x(t),t)$ breaks down into n-elements $S_i(x_i(t),t)$, $i=\overline{1n}$, because it is reciprocal relationships $a_{ij}(t)$, $i,j=1,2,...n$ that provide system acquisition of highly-integrated emergence properties. Reciprocal relations between the components of the vector x_i can also occur due to other off-diagonal elements: $b_{ij} \in B$ and $g_{ij} \in G$ that are also subjected to regulation. However, it is possible to consider the system model (1), where the matrix of the state $A(t)$ is complete, and the matrices $B(t)$ and $G(t)$ are diagonal. This assumption is justified by the fact that under fairly general restrictions due to the matrix sin/cos conversion, the system (1) is brought to such form.

There are other options for presenting models of dynamic systems. Thus in multi-agent networks [5, 6], which also include telecommunications networks we usually consider a basic differential model in the following form:

$$dx_i(t)/dt = -\sum_{j=1}^{n} \alpha_{ij}(t)(x_i(t)-x_j(t)),\ i=1,2,...,n. \tag{3}$$

Equation (3) can be interpreted as follows. The condition of i th node $x_i(t)$ is characterized by a vector with components $x_{ij}(t)$, where i is a node number, j is numbers of adjacent nodes. Components $x_{ij}(t)$ determine the state of the directions from node i to node j.

Besides it is not obvious how we can directly apply base differential model (3), primarily due to uncertainty caused by either the difference between the states $x_i(t)-x_j(t)$ or models of these states. Let us add the physical sense to the components of the difference. Obviously, $x_i(t)$ and $x_j(t)$ are vectors generated respectively in i th and j th nodes and their condition may be accurately known to the model (1). The state of the components of adjacent nodes $x_j(t)=\{x_{ji}(t)\}$ are the data coming from the lines from all j nodes to the given i th node. This vector $x_j(t)$ is subjected to observation (measurement), so regarding it the model (2) must be applied, where $v_j(t)$ is the noise in the measurement channel, counter error, other factors affecting the accuracy of measurement. Besides another difference (residual) can be more realistic the form of which involves different observed measurement vector $y_j(t)$, which corresponds to the practical content of the problem. The very task becomes the problem of stochastic approximation [9], which is widely used in modern communication technologies. It is used for example in determining the round-trip time RTT, in the technology of the TCP algorithms RED-overload prevention router buffers etc. [8]. It is known [8, 9] that the evaluation obtained by the method of stochastic approximation is optimal for the criterion of minimum mean-square deviation (MMSD).

Thus the problem (3) can be transformed into the following:

$$dx_i(t)/dt = -\sum_{j=1}^{n}\alpha_{ij}(t)(x_{ij}(t) - y_{ji}(t)), \qquad (4)$$

where the difference $x_{ij}(t) - y_{ji}(t)$ is the essence, the discrepancy. Under optimal procedures the discrepancy has the nature of innovation process such as white noise. Moreover, the very discrepancy $x_{ij}(t) - y_{ji}(t)$ usually serves as a control signal in assessing the state under the implementation of control algorithms for supervision and the management of the system state [8].

In (4) we need to select relationship coefficients α_{ij} in order to optimize the system: to minimize the estimation error, to ensure an acceptable rate of convergence procedure (4) in quasi-stationary regions and thereby obtain the greatest impact from the system consideration including relationships when considering the dynamics.

In this case (4) it is not so important whether the system $S(x_n(t), t)$ is centralized or decentralized. The relevant interactions of α_{ij} that determine its connectivity take place for each of these structures. Connectivity or structural connectivity is an essential feature of any system, because when the connection disappears the system disappears as well.

We should also note the possibility of a false relationship between the components of the vector x_i, which can occur even at zero values of the off-diagonal α_{ij}, b_{ij}, c_{ij}. This reciprocal connection between the independent components x_i and x_j can occur due to the appearance of off-diagonal components of the measurements matrix $H(k) = [h_{ji}]$ in equation (2). Such a situation arises when during the measurement of one state x_{ij} the active measuring channel receives transient signals from the other x_{ji}, i.e. when there are transient cross- interference. This connection is artificially produced by the appearance of components $h_{ji} \neq 0$ and it is usually undesirable. Eliminating of these unwanted connections is usually carried out by engineering methods.

At the other extreme case under maximum connectivity matrix $\alpha_{ij} \in A$ may be full. In practice this may mean that all nodes connected to each other. Obviously in dynamical systems, including telecommunications such a full mesh case, it is not real, because in practice communication of all the nodes with all the nodes at the same time is not necessary as a rule. The case when the matrix A_{ij} is very sparse will be more realistic.

II. Assessment of connected systems state

Procedure (4) is usually implemented in discrete form:

$$x_i(k+1) = x_i(k) + \sum_{j=1}^{n} \alpha_{ij}(k)\left[x_i(k) - y_i(k)\right], \tag{5}$$

where $y_j(k) = H(k)x_j(k) + v_j(k)$ is the system of equations vector state observation.

When analyzing the structure connectivity in a multi-dimensional dynamic system not only rated values α_{ij} are important, but also the number of off-diagonal elements. From the works of Gardner, Ashby [7] and others it is known that a certain number of off-diagonal elements (for complex 10-dimensional differential system) cause unstable regime. It is stated that such a strongly connected model diverges under $k \to \infty$. Here [7] data of the existence of a critical connectivity, when the number of off-diagonal elements reaches 13 % of the maximum possible number $n \times n$ is presented. In this case, the instability of the model is specific for both linear and non-linear dynamic systems. Here are the characteristic graphs of stability depending on the connectivity [7] (Fig. 1).

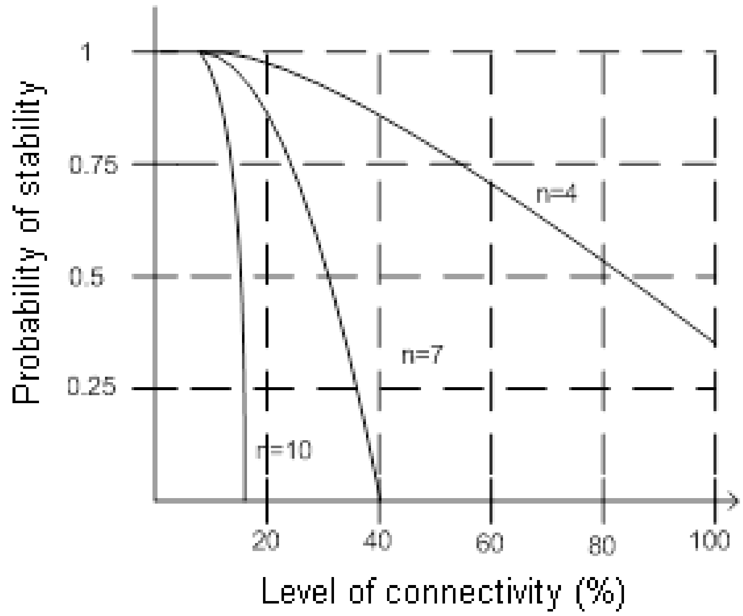

Fig. 1. Graph for probability of stability in n-dimensional model
based on the level of connectivity

In the scientific and technical literature, much attention is paid to the recursive filtering procedures, the determination of estimates, statistical inference, etc. [8, 9]. Univariate procedure (1) and (4) are fairly well studied [8, 9] and their stability is ensured by an appropriate choice of the constant step α_{ij} under rather general restrictions on the properties of the statistics.

However, quite often we have to solve the problem of finding the evaluation of two- or more dimensional processes $x^T(k) = (x_1, x_2, \ldots x_n), n = 2,3,\ldots n,$ that are functionally or statistically connected. As a real example of processes connectivity it is possible to indicate processes of network management, the signals in the adjacent antennas in MIMO technology, processes in the interacting routers, etc. When such bonds increases supercritical, stability is expected to be lost, at the same time under weak connections it is expected, that the estimation accuracy of each component x_i should increase, because uncertainty in the independent n-dimensional system is higher than in the presence of specified dependencies between the variables in the system. There is a question of how does the reciprocal information become useful, and under what parameters of the recursive procedure (1) or (5), in particular in the presence of non-zero values, gain in accuracy of estimation. Another important task is to answer the question of what is the best way to use the reciprocal information defined by the presence of $\alpha_{ij} \neq 0$.

It is known [9] that the recursive methods can produce quite consistent estimate by the method of stochastic approximation for sufficiently slowly changing processes $x(k)$. Thus, obviously the chosen sampling step $\Delta t = (k+1) - k$ should be much smaller than the correlation interval or quasi-period process.

III. The results of computer simulation

Selection of the procedure. The optimal procedure for the assessment of random processes is the Kalman-Bucy filter (FKB) [8], however, we often use a simpler procedure of stochastic approximation (SA) [9]:

$$x(k+1) = x(k)(1-\beta) + \beta y(k), \tag{6}$$

SA has been transformed form the classical procedure

$$\hat{x}(k+1) = \hat{x}(k) - a(\hat{x}(k) - y(k)). \tag{7}$$

It is known [8, 9] that SA procedure (7) is optimal for estimating random variables x. Its use for the evaluation of random processes lead to the fact that after each change $x(t)$ the procedure is undergoing the transition mode. In other words, under the evaluation of the random process SA procedure is in continuously transition state, which leads to errors in estimation. However, these errors can be significantly reduced by reducing the sampling step $\Delta t = (k+1) - k$ comparatively to a random process correlation interval $\tau_{кор}$. Practice shows that under the length of the steps $\Delta t = (0{,}01...0{,}001)\tau_{кор}$ the influence of the transition process is less noticeable.

To study the effect of reciprocal connections a_{ij} it is enough to choose 2-dimensional procedure:

$$\begin{cases} \hat{x}_1(k+1) = \hat{x}_1(k) + a_{11}(\hat{x}_1(k) - y_1(k)) + a_{12}(\hat{x}_2(k) - y_1(k)); \\ \hat{x}_2(k+1) = \hat{x}_2(k) + a_{12}(\hat{x}_2(k) - y_2(k)) + a_{22}(\hat{x}_1(k) - y_2(k)), \end{cases} \tag{8}$$

where the values $a_{ii} = const$, a_{jj} - changes from 0 to a_{ij} and the impact of these dependencies should be investigated . Block diagram (8) is shown in Figure 2.

Selection of the estimated functions. We present the estimable functions as two sinusoidal functions $x_i(k) = u_i \cos\varphi_i(k)$ with the same amplitudes $u_1 = u_2$ with different initial phases and step $\Delta\varphi_i = \varphi_i(k+1) - \varphi_i(k) = 4^\circ$. Observation equation is taken in the form:

$$y_i(k) = x_i(k) + v_j(k), \tag{9}$$

where $v(k)$ is the sampling of a Gaussian white noise with power spectral density $N_v(k)$. Signal-to-Noise ratio: P_c / N_v is 5dB.

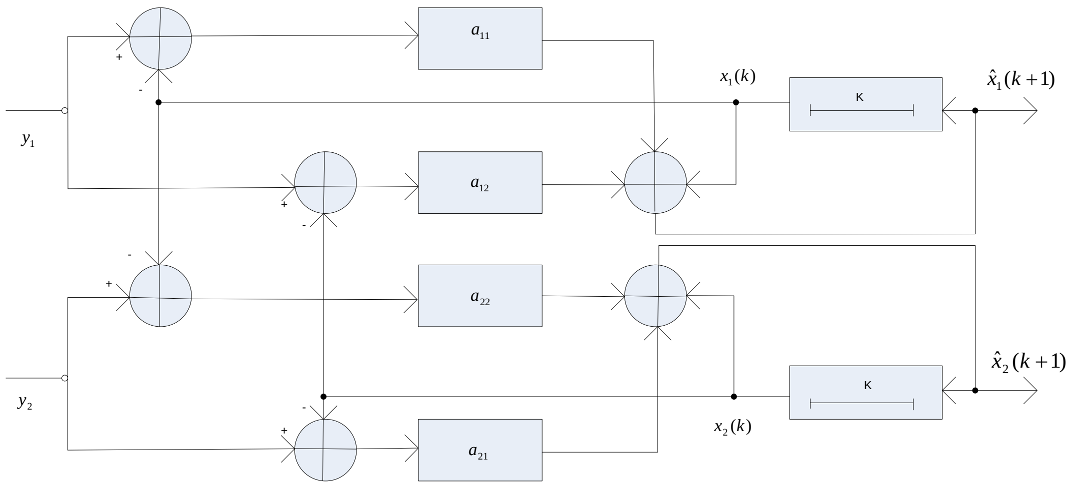

Fig. 2. Block diagram of the two-dimensional algorithm for evaluation process $x_i(k)$

We take a sample normalized value of the a posteriori variance σ_{xi}^2 as function indicator calculated by the formula:

$$\sigma_{xi}^2 = \frac{1}{K-1}\sum_{k=1}^{K}(x_i(k) - \hat{x}_i(k))^2, \tag{10}$$

where K - the size of the sample, amounting to 10000 counts.

Discussion of the results. Figure 3 shows three different graphs of change in estimation accuracy for the sample satisfying equation (6) depending on the number of sampling steps, counted from the moment of switching procedures and different coefficient values a_{ii} and a_{ij}. Graph № 1 is given for comparison. It describes the potential abilities of SA for the evaluation procedure of the random variable $x = const$. The graph shows that with the increase of the number of steps $k > (50...100)$ the a posterior variance decreases asymptotically to zero. This means that the random variable x, observed on the background of the Gaussian white noise, can be accurately evaluated asymptotically: $\sigma_x^2(k) \to 0$ under $k \to \infty$. Graphs № 2 and № 3 show similar characteristics to the previous, but the asymptotic zero of a posteriori dispersion at $k \to \infty$ is not observed. There is a residual value σ_{xi}^2 that is caused by the influence of unsteady transients. The main result of the experiment is that the a posteriori dispersion σ_{xi}^2 for the connected processes is lower at $(5...10)\%$ than that one for independent processes under other equal conditions. Accordingly, the accuracy of the estimate for the connected processes is higher.

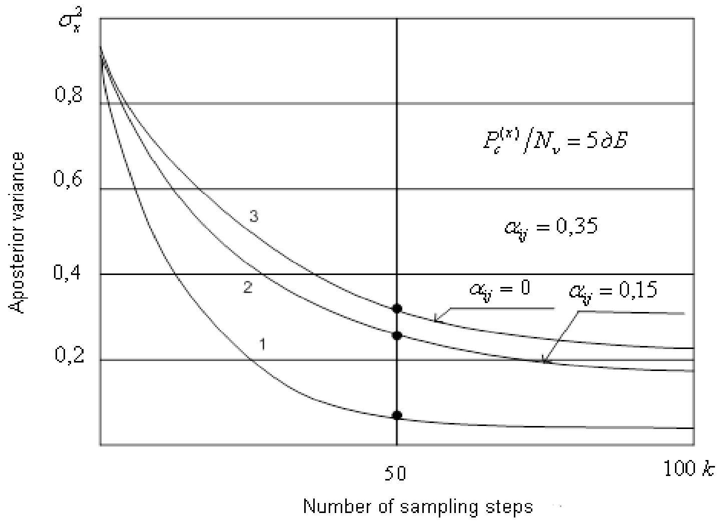

Fig. 3. Graphs of changes for a posteriori variance estimation, estimation errors $\tilde{x}_k = (\hat{x}_k - x_k)$ under signal/noise ratio $P_C^{(x)}/N_\nu = 5$ dB. Estimation error of DC component - $1, \alpha_{ij} = 0;$ sinusoidal functions - 2, $\alpha_{ij} = 0{,}15$; $-3, \alpha_{ij} = 0$.

Figure 4 shows graphs for a posteriori dispersion depending on the magnitude of the coupling coefficient a_{ij}. Graph confirms the previous conclusion, and shows that with increasing level of connectivity accuracy for the estimation grows. These studies have used relatively weak connections. With the increase in the level of connectivity and the number of these connections definitive conclusions cannot be done.

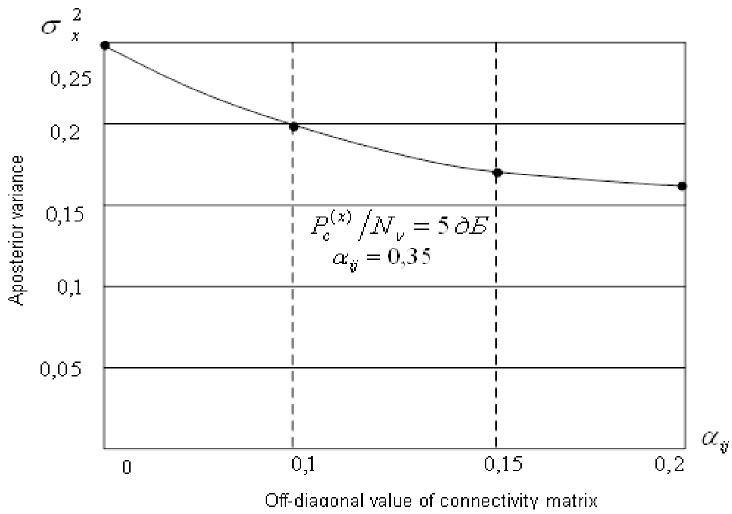

Fig. 4. Graph of changes for a posteriori variance error estimation for different off-diagonal value α_{ij} of the connectivity matrix

Conclusion

1. The presence of the connectivity between the elements of systems provides its acquisition of new highly-integrated properties: consistency, integrity, emergence. The presence of connectivity in the network structures, such as infocommunication systems, provides the acquisition of properties for the reliability and survivability of networks.

2. For fixed (invariable with time parameters) networks connection is displayed as connectivity matrix, which may have a binary structure or consist of quantitative data defining the level of the connection (probability, number of channels, distance, etc.). The coefficient of connectivity or connectivity probability for such networks is found by the approximate methods of Ezary-Proschan, Polesski etc.

3. For dynamic (with time-varying elements of connection) network connectivity values can be obtained through the current assessment of the state of connectivity for each node from all adjacent nodes. For real posteriori evaluation of connectivity it is appropriate to use the state-space model that allows us to characterize the dynamic systems as deterministic and stochastic. Practice has shown that the state of the process allows to use the methods of stochastic approximation under the condition of choosing the sampling rate $\Delta t << \tau_{кор}$.

4. The analysis of multi-dimensional differential systems shows that the existence of connections between components of the system cannot have arbitrary values. A large number of connections leads to unstable regimes. In the other extreme case: under the absence of reciprocal connection the system loses its system properties (integrity, emergence).

5. The connection between random processes improves accuracy (reduced values of a posterior variance) of the estimates of the components for these processes compared to that case when data of the process are independent.

References

1. Лекции по теории графов / В.А. Емеличев, О.И. Мелаников, В.И. Сарванов, Р.И. Тышкевич. – М.: Наука, 1990. – 384 с.

2. *Esazy J., Proshkan F.*, Coherent Structures of Non-Idential Components // Technometrics. – 1963. – Vol. 5, № 2. – P. 191-309.

3. *Полесский В.П.* Оценки вероятности связности случайного графа // Проблемы передачи информации. – 1990. – Т. 26, Вып. 1. – С. 90-101.

4. *Полесский В.П.* Схема вычисления приближенного значения характеристик связности в вероятностной модели устойчивости системы // Техника средств связи. Серия «Системы связи». – 1986. – Вып. 6. – С. 34-41.

5. *Чеботарев П.Ю., Агаев Р.П.* Согласование характеристик в многоагентных системах и спектры лапласовских матриц орграфов // Автоматика и телемеханика. – 2009. – №3 – С. 136-152.

6. *Джунсов И.А., Фрадков А.Л.* Синхронизация в сетях линейных агентов с обратными связями по выходам // Автоматика и телемеханика. – 2011. – № 8. – С. 41-52.

7. *Gardner M.R., Assby W.R.* Connectauce of large dynamic (cybernetic) systems: Critical values stability // Nature. – 1970. – № 228. – P. 784.

8. *Popovskij V., Barkalov A., Titarenko L.* Control and Adaptation in Telecommunication Systems. Mathematical Foundations. – Berlin: Springer-Verlag, 2011. – 173 p.

9. *Невельсон М.Б., Хасьминский Р.З.* Стохастическая аппроксимация и рекуррентное оценивание. – М.: Наука, 1972. – 298 с.