

## Identification of preferences in decision support systems

V. Beskorovainyi, H. Berezovskyi

Kharkiv National University of Radio Electronics,  
Nauka Ave, 14, Kharkov, 61166, Ukraine  
vladimir.beskorovainyi@nure.ua

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**Abstract.** In this work we obtained the solution of increasing of adequacy of models of multicriteria evaluation for the project and management decision support systems. Modifications of the utility functions of partial criteria and the procedure for calculating their values are proposed, which makes it possible to improve the accuracy of approximation of the preferences of the person making the decision, as well as significantly reduce the time for calculating their values. For the parametric synthesis of universal multicriteria estimation models based on the Kolmogorov-Gabor polynomial, an improvement in the method of comparator identification by calculating the Chebyshev point and the discrepancy vector is proposed.

**Key words:** decision support, multi-criteria optimization, utility function of partial criteria.

### INTRODUCTION

One of the most important tasks of modeling intellectual activity is the task of studying the processes of multifactorial evaluation and decision-making by man. Formalization of these processes allows to improve existing and create new intellectual decision support systems (DSS). This contributes to improve the quality of decisions in control systems and automated design of anthropogenic objects systems [1-3]. This systems are important in the design and management of complex large-scale objects [4-6]. Using this systems in selecting of effective design and management solutions allows to significantly reduce the cost of the creation and operation of such facilities, providing the required levels of their functional characteristics [7-8].

At the heart of modern models of decision-making lies the paradigm of maximizing utility [9]. It is believed that the person making the decision (DM), when choosing options from a set of allowable  $x \in X$  ascribes to them some utility  $P(x)$ , the quantitative values of which determine its choice:

$$x^o = \arg \max_{x \in X} P(x). \quad (1)$$

General utility functions (GUF)  $P(x)$  are formed on the basis of the utility functions of the partial criteria (UFPC)  $\xi_i[k_i(x)] = \xi_i(x)$ ,  $i = \overline{1, m}$  (where  $m$  is the number of partial criteria  $k_i(x)$ ). The synthesis of the GUF  $P(x)$  is reduced to solve a set of problems of structural and parametric identification.

In the general case, in the process of identifying the preferences of DM, it is necessary to solve questions related to the choice of similarity criteria, input signals, structure and parameters of the model, an estimation of its accuracy and adequacy [10]. The most interesting, both theoretically and in practice, are the problems of choosing the structure of a function  $P(x)$  and its parameters (parameters of utility functions of partial criteria and their weight coefficients) [9].

### ANALYSIS OF THE MODERN CONDITION OF THE PROBLEM

The methodology of modern DSS is based on the theory of multicriteria decision making [1-3, 9-10]. The choice of the best solution from the set of effective ones only in the simplest situations can be carried out by a decision maker without the use of formal methods [11]. To automate the procedures for evaluating multicriteria solutions, it is necessary to involve additional information on the value of individual formalized properties (partial criteria) and their meanings. The most important task of formalizing the decision-making during of multicriteria optimization is to determine the metric for ranking options [9]. As a methodological basis for constructing a metric, the utility theory is traditionally used, according to which for each of the variants from the admissible set the value of its utility can be determined  $P[\xi(x), \lambda]$  (where:  $\xi(x) = [\xi_i(x)]$   $i = \overline{1, m}$  – vector of UFPC  $\xi = [\xi_i]$ ,  $i = \overline{1, m}$ ;  $\lambda = [\lambda_i]$ ,  $i = \overline{1, m}$  – vector of weight coefficients of partial criteria  $k_i(x)$ ,  $i = \overline{1, m}$ ). It is considered that for all  $x, y \in X$ :

$$x \sqcup y \leftrightarrow P[\xi(x), \lambda] = P[\xi(y), \lambda]; \quad (2)$$

$$x \succ y \leftrightarrow P[\xi(x), \lambda] > P[\xi(y), \lambda]; \quad (3)$$

$$x \sqsupseteq y \leftrightarrow P[\xi(x), \lambda] \geq P[\xi(y), \lambda]. \quad (4)$$

In view of the incomplete certainty of the requirements to the properties of solutions as a function of general utility  $P[\xi(x), \lambda]$  it is proposed to use the membership function to the fuzzy set "best option". In this case, the fuzzy set "best option" can be represented as a set of ordered pairs [12]:

$$\langle \text{Best option} \rangle = \{ \langle x, P[\xi(x), \lambda] \rangle \},$$

where:  $x \in X$  – a variant of the set of admissible;  $P[\xi(x), \lambda]$  – the degree of membership of

the variant to the "best option" fuzzy.

The definition of the metric for ranking options  $x \in X$  is the solution of the task of identification of preferences and reduces to solving problems of structural and parametric synthesis of the function  $P[\xi(x), \lambda]$  [13]. In general, it involves choosing the type and parameters of the functions  $\xi_i(x)$ ,  $i = \overline{1, m}$ ,  $P[\xi(x), \lambda]$  and the vector of weight coefficients  $\lambda = [\lambda_i]$ ,  $i = \overline{1, m}$ .

As the criteria of identification (similarity of models), depending on the conditions of the problem, a minimum of the total (mean, maximum, total quadratic) absolute, relative error of the estimation of the general utility  $P[\xi(x), \lambda]$ , the maximum strength of preferences, the midpoint, the maximum of the correctness of choice, or the minimum of the error of restoring order on subsets of admissible variants  $X' \subseteq X = \{x\}$  [10].

UFPC  $\xi_i(x)$ ,  $i = \overline{1, m}$  in this case are considered as a membership function to a diffuse set of "best option" according to partial criteria  $k_i(x)$ ,  $i = \overline{1, m}$ . They implement mappings  $\xi_i: k_i(x) \rightarrow E^1$ ,  $i = \overline{1, m}$  and should be universal, well-adapted to take into account the characteristics of specific situations of multi-criteria choice. They are presented with a number of requirements [9]: monotonicity and dimensionlessness; a single change interval (from 0 to 1); invariance to the form of extremum of a particular criterion (*min* or *max*); The possibility of mapping linear and non-linear dependencies on the values of a partial criterion.

The greatest distribution in the practice of multicriteria optimization was obtained by the membership functions of the form [9]:

$$\xi_i(x) = \left( \frac{k_i(x) - k_i^-}{k_i^+ - k_i^-} \right)^{\alpha_i}, \quad i = \overline{1, m}, \quad (5)$$

where:  $k_i(x)$ ,  $k_i^+$ ,  $k_i^-$  – the value of the  $i$ -th partial criterion for variant  $x$ , the best and the worst values of the  $i$ -th criterion,  $i = \overline{1, m}$ ;  $\alpha_i$  – parameter defining the form of dependence ( $\alpha_i = 1$  – linear,  $0 < \alpha_i < 1$  – concave,  $\alpha_i > 1$  – convex).

The disadvantage of functions of the form (5) is the impossibility of realizing S- and Z-shaped dependencies on the values of the partial criterion. Free from this drawback are the gluing of power functions from [12], the Gaussian function, Harrington function, the logistic function, and their modifications [14]. However, computer procedures for calculating their values have a sufficiently high temporal complexity.

The most universal among the functions for multifactor estimation is a function constructed on the basis of the Kolmogorov-Gabor polynomial [9]:

$$P[\xi(x), \lambda] = \sum_{i=1}^m \lambda_i \xi_i(x) + \sum_{i=1}^m \sum_{j=i}^m \lambda_{ij} \xi_i(x) \xi_j(x) + \dots + \sum_{i=1}^m \sum_{j=i}^m \sum_{l=j}^m \lambda_{ijl} \xi_i(x) \xi_j(x) \xi_l(x) + \dots, \quad (6)$$

where:  $\lambda_i, \lambda_{ij}, \lambda_{ijl}$  – weight coefficient of partial criteria and their product;  $\xi_i(x), \xi_j(x), \xi_l(x)$  – UFPC's  $k_i(x), k_j(x), \dots, k_l(x)$ .

If a vector of parameters  $\lambda$  is defined and the form of the utility functions of the partial criteria is known  $\xi_i(x)$ ,  $i = \overline{1, m}$ , then the problem of choosing the best variant for models of the form (6) can be reduced to an optimization problem of the form (1).

The task of determining the vector of weighting coefficients  $\lambda$  for models of the form (6) it was traditionally solved by expert methods by ranking methods, assigning points, consecutive preferences, pair comparisons [11]. The disadvantages of these methods are the complexity and relatively low accuracy of estimates. As an alternative to expert estimation of parameters, the technology of comparator identification is increasingly being used [9-10, 14-15].

The review of the current state of the problem of identification of preferences of decision-makers in DSS shows that by now it is far from its solution and requires further research. The questions of estimating the time complexity of procedures for calculating the values of UFPC and improving them in the direction of reducing the time for calculating their values remain practically unexplored. The theory of structurally-parametric identification of preferences of decision-makers with the use of universal models of multicriteria estimation requires further development.

## OBJECTIVES

The aim of the research is to increase the effectiveness of multi-criteria evaluation procedures in decision support systems.

To achieve this goal, it is necessary:

- to develop a procedure for the formation of monotonic membership functions for fuzzy sets "best option" by partial criteria, reducing the time complexity of calculating their values;
- to develop a method of parametric synthesis of universal models of multicriteria estimation and choice of decisions;
- to carry out a comparative analysis of the time complexity and accuracy of approximation of the preferences of the decision-maker using universal models of multifactor estimation and decision-making.

## FORMALIZATION OF PREFERENCES OF DM UNDER THE VALUE OF PARTIAL CRITERIA

To simplify the universal membership functions of fuzzy sets "best option" we use the procedure of linear normalization of the partial criteria (5) for  $\alpha_i = 1$ :

$$\bar{k}_i(x) = \frac{k_i(x) - k_i^-}{k_i^+ - k_i^-}, \quad i = \overline{1, m}. \quad (7)$$

This without loss of accuracy will simplify the universal membership functions, which allow to realize linear and nonlinear (including S- and Z-shaped) dependencies on the values of partial criteria.

Taking into account the normalization (7), the most commonly used membership functions [10, 12, 14] will have the following form:

– Gaussian function [14]:

$$\xi(x) = \exp\left[-\frac{(\bar{k}(x) - 1)^2}{c}\right], \quad (8)$$

where:  $c > 0$  – a parameter that defines a particular type of dependency;

– logistic function [14]:

$$\xi(x) = \frac{1}{1 + \exp\left[-\frac{(\bar{k}(x) - a)}{b}\right]}, \quad (9)$$

where:  $a$  – abscissa of inflection point;  $b$  – parameter that defines a particular type of dependency;

– Harrington function [14]:

$$\xi(x) = \exp\left\{-\exp\left[(g \cdot \bar{k}(x) - a)\right]\right\}, \quad (10)$$

where:  $g$  – nonlinearity parameter;  $a/g$  – determines the inflection point;

– modified Gaussian function [14]:

$$\xi(x) = \exp\left[-\frac{(\bar{k}(x) - 1)^{2\alpha}}{c}\right], \quad (11)$$

where:  $c > 0$  – parameter that defines a particular type of dependency;  $\alpha$  – nonlinearity parameter

– gluing function of power functions [14]:

$$\xi(x) = \begin{cases} \bar{a} \cdot \left(\frac{\bar{k}(x)}{\bar{k}_a}\right)^{\alpha_1}, & 0 \leq \bar{k}(x) \leq \bar{k}_a; \\ \bar{a} + (1 - \bar{a}) \cdot \left(\frac{\bar{k}(x) - \bar{k}_a}{1 - \bar{k}_a}\right)^{\alpha_2}, & \bar{k}_a < \bar{k}(x) \leq 1, \end{cases} \quad (12)$$

where:  $\bar{k}_a, \bar{a}$  – normalized values of the coordinates of the point of gluing the function,  $0 \leq \bar{k}_a \leq 1, 0 \leq \bar{a} \leq 1$ ;  $\alpha_1, \alpha_2$  – coefficients that determine the form of the dependence on the initial and final segments of the function;

– gluing function of power functions, built on the basis of a function [12]:

$$\xi(x) = \begin{cases} 2^{p-1} \cdot [\bar{k}(x)]^p, & 0 \leq \bar{k}(x) \leq 0.5; \\ 1 - 2^{p-1} \cdot \left[\frac{0.5 - \bar{k}(x)}{0.5}\right]^p, & 0.5 < \bar{k}(x) \leq 1, \end{cases} \quad (13)$$

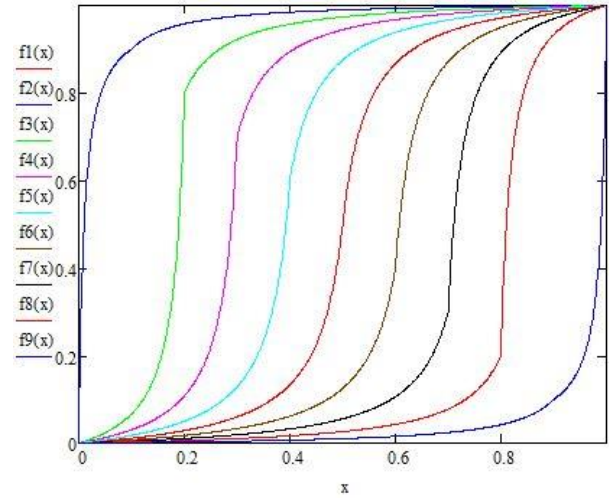
where:  $p$  is a parameter that determines the form of dependence.

Functions (8) – (13) strongly change their values at the entrance to the dead zones (approaching the partial characteristics of the variants to the worst and best values  $\bar{k}(x) \rightarrow 0$  и  $\bar{k}(x) \rightarrow 1$ ). This can lead to significant errors in determining the properties of the variants according to partial criteria and have a significant effect on the error in calculating of the general estimation  $P[\xi(x), \lambda]$  (6).

To overcome these shortcomings, a modification of the gluing function (12) with a smaller dead zone (Fig. 1) is proposed:

$$\xi(x) = \begin{cases} \bar{a} \cdot (b_1 + 1) \cdot \left(1 - \left(b_1 / \left(b_1 + \frac{\bar{k}(x)}{\bar{k}_a}\right)\right)\right), & 0 \leq \bar{k}(x) \leq \bar{k}_a; \\ \bar{a} + (1 - \bar{a}) \cdot (b_2 + 1) \times \\ \times \left(1 - \left(b_2 / \left(b_2 + \frac{\bar{k}(x) - \bar{k}_a}{1 - \bar{k}_a}\right)\right)\right), & \bar{k}_a < \bar{k}(x) \leq 1, \end{cases} \quad (14)$$

where:  $b_1, b_2$  – parameters that determine the form of the dependence on the initial and final segments of the function.



**Fig. 1.** The type of UFPC (14) for different values of the parameters  $\bar{k}_a, \bar{a}, b_1, b_2$

It is established that the accuracy of approximating the preferences of the decision-maker with the help of the function (12) and the proposed modification (14) is several times higher than using the functions (8) - (11) and (13). In this case, the procedures for calculating the values of the functions (13) and (14) have much less time complexity than the values of the functions (8) - (11) and (13).

To further reduce the time complexity of the procedures for calculating the values of the function (14), its piecewise linear approximation is performed with a uniform approximation on the segments  $[0, \bar{k}_1], [\bar{k}_1, \bar{k}_2], \dots, [\bar{k}_{n-1}, 1]$ :

$$\left\{ \begin{array}{l} \int_0^{\bar{k}_1} |\xi(x) - d_1 \cdot \bar{k}(x) - h_1| dk = \int_{\bar{k}_1}^{\bar{k}_2} |\xi(x) - d_2 \cdot \bar{k}(x) - h_2| dk; \\ \dots\dots\dots \\ \int_{\bar{k}_{n-2}}^{\bar{k}_{n-1}} |\xi(x) - d_{n-1} \cdot \bar{k}(x) - h_{n-1}| dk = \int_{\bar{k}_{n-1}}^I |\xi(x) - d_n \cdot \bar{k}(x) - h_n| dk, \end{array} \right. \quad (15)$$

where:  $d_i, h_i, i = \overline{1, n}$  – scaled for the  $i$ -th interval parameters of linear functions.

The solution of the system of integral equations (15) makes it possible to determine the best coordinates of the nodes  $h_i, \bar{k}_i, i = \overline{1, n}$  and parameters  $h_i, h_i, i = \overline{1, n}$  for piecewise-linear approximation of the function (14):

$$\xi(x) = \begin{cases} d_1 \cdot \bar{k}(x) + h_1, & 0 \leq \bar{k}(x) \leq \bar{k}_1; \\ d_2 \cdot \bar{k}(x) + h_2, & \bar{k}_1 \leq \bar{k}(x) \leq \bar{k}_2; \\ \dots\dots\dots \\ d_n \cdot \bar{k}(x) + h_n, & \bar{k}_n \leq \bar{k}(x) \leq I. \end{cases} \quad (16)$$

To reduce the number of operations required for calculating function values  $\xi(x)$ , it is proposed to use a single preliminary calculation of their parts that do not change when the values of the particular criterion  $\bar{k}(x)$  change. This allows to reduce the calculation time for function values by 25,9%.

The accuracy of the approximation obtained from relation (15) increases with the number of nodes, but the number of computer operations for calculating the values of the function increases. When approximating the preferences of individual DM with the use of four nodes, the required order of accuracy is preserved, and the calculation time of the function values is reduced by 19,7%.

#### PARAMETRIC SYNTHESIS OF THE MODEL OF MULTI-CRITERIAL ESTIMATION

The parametric synthesis problem is considered for the multicriterion estimation model (6). As its components, the universal utility function  $\xi_i(x), i = \overline{1, m}$  (14) of partial criteria is used, allowing to realize linear, convex, concave, S- and Z-shaped dependences on the values of partial criteria  $k_i(x), i = \overline{1, m}$ .

To solve the problem, we use the technology of comparator identification, the essence of which is the following [9-10]. A subset of the set of admissible variants is given  $X' \subseteq X$  and the corresponding values of partial criteria  $k_i(x), i = \overline{1, m}$ .

On it it is necessary to allocate a subset of Pareto-optimal variants  $X^C \subseteq X'$ . DM analyzes pairs of options  $x, y \in X^C$ , which form in his mind some subjective assessments of utility  $P[\xi(x), \lambda]$  and  $P[\xi(y), \lambda]$ , whose values can not be measured. On the basis of these assessments, the DM gives an opinion on the equivalence or preferences of the variants (forms binary equivalence relations, strict or non-strict preferences):

$$\begin{aligned} -R_E(X^C) &= \{ \langle x, y \rangle : x, y \in X^C, x \sim y \}; \\ -R_S(X^C) &= \{ \langle x, y \rangle : x, y \in X^C, x \succ y \}; \\ -R_N(X^C) &= \{ \langle x, y \rangle : x, y \in X^C, x \succ y \}. \end{aligned}$$

For them, the corresponding systems of equations and inequalities are composed:

$$P[\xi(x), \lambda] = P[\xi(y), \lambda], \langle x, y \rangle \in R_E(X^C), \quad (17)$$

$$P[\xi(x), \lambda] > P[\xi(y), \lambda], \langle x, y \rangle \in R_S(X^C), \quad (18)$$

$$P[\xi(x), \lambda] \geq P[\xi(y), \lambda], \langle x, y \rangle \in R_N(X^C), \quad (19)$$

where:  $\lambda$  – the required vector of the GUF parameters.

The problem of parametric identification of the GUF is reduced to the determination of the vector  $\lambda = [\lambda_i]_{i=1}^N$  (where  $N$  – number of model parameters), satisfying the system of equations and inequalities (17), (18) or (19). In this case, the generated systems of inequalities or equations can be inconsistent or have an infinite number of solutions.

Let's choose as a criterion for identifying preferences of the DMs, the minimum of the error of recovery of the order of the preferences of the variants and the minimum of the sum of the squares of the error of estimates of the utility of the variants. The number of terms of the model (6) is chosen proceeding from the required accuracy of restoring the preferences of the DM, the dimension of the problem and the available computing resources. The maximum number of terms is  $N = C_{m+n}^n - 1$  (where  $m$  is number of partial criteria;  $n$  is power of polynomial model).

Let's introduce the following notation:

$$\begin{aligned} \xi_i(x) \cdot \xi_l(x) &= \xi_{m+l}(x), \lambda_{1,1} = \lambda_{m+1}, \\ \xi_j(x) \cdot \xi_2(x) &= \xi_{m+2}(x), \lambda_{1,2} = \lambda_{m+2}, \dots \end{aligned} \quad (20)$$

Taking into account the introduced notation (20), the function (6) can be represented in the additive form:

$$P[\xi(x), \lambda] = \sum_{i=1}^N \lambda_i \xi_i(x). \quad (21)$$

The requirement that pairs of variants belong to a subset of Pareto optimal  $x, y \in X^C$ , is due to the fact that taking into account the dominant variants from the subset of the consent  $X^S = X \setminus X^C$  when forming binary relations strictly  $R_S(X)$  and nonstrict  $R_N(X)$  preferences does not bear useful information, i.e.  $x \succ y \forall x \in X^C$  and  $\forall y \in X^S$ . This is a consequence of the fact that relations of strictly and unstrict preferences for dominant variants are fulfilled for any values of weighting coefficients  $\lambda_i, i = \overline{1, N}$ .

From the equivalence relation  $R_E(X^C)$  for the model (21) from condition (17) we obtain a system including  $n_E$  equations:

$$\left\{ \begin{array}{l} \eta_j(\lambda) \equiv \sum_{i=1}^N \lambda_i \xi_i(x) = \sum_{i=1}^N \lambda_i \xi_i(y); \\ \langle x, y \rangle \in R_E(X^C), \quad j = \overline{1, n_E}, \end{array} \right. \quad (22)$$

and also the equation for normalizing the weight vector of partial criteria:

$$\eta_{n_E+1}(\lambda) \equiv \sum_{i=1}^N \lambda_i = 1, \quad \lambda_i \geq 0, \quad i = \overline{1, N},$$

where:  $n_E = \text{Card } R_E(X^C)$  – equivalence power  $R_E(X^C)$ .

From relations of strict  $R_S(X^C)$  and nonstrict  $R_N(X^C)$  preferences we obtain systems of nonlinear inequalities and normalizing conditions:

$$\left\{ \begin{array}{l} \eta_j(\lambda) \equiv \sum_{i=1}^N \lambda_i \xi_i(x) > \sum_{i=1}^N \lambda_i \xi_i(y); \\ \langle x, y \rangle \in R_S(X^C), \quad j = \overline{1, n_S}; \\ \eta_{n_S+1}(\lambda) \equiv \sum_{i=1}^N \lambda_i = 1, \quad \lambda_i \geq 0, \quad i = \overline{1, N}; \end{array} \right. \quad (23)$$

$$\left\{ \begin{array}{l} \eta_j(\lambda) \equiv \sum_{i=1}^N \lambda_i \xi_i(x) > \sum_{i=1}^N \lambda_i \xi_i(y); \\ \langle x, y \rangle \in R_N(X^C), \quad j = \overline{1, n_N}; \\ \eta_{n_N+1}(\lambda) \equiv \sum_{i=1}^N \lambda_i = 1, \quad \lambda_i \geq 0, \quad i = \overline{1, N}, \end{array} \right. \quad (24)$$

where:  $n_S = \text{Card } R_S(X^C)$ ,  $n_N = \text{Card } R_N(X^C)$  – power relationship's  $R_S(X^C)$  and  $R_N(X^C)$ .

The resulting systems of equations and inequalities (22) - (24) are homogeneous and define sets of planes passing through the origin. The second part of them in the form of normalizing conditions  $\sum_{i=1}^N \lambda_i = 1, \lambda_i \geq 0, i = \overline{1, N}$  define secants.

One way to solve such systems is to search for the Chebyshev point [9-10]. It allows us to reduce the initial problems to problems of linear programming. To do this, we introduce an additional variable  $\lambda_{N+1}$  into the system of equations (22) for the resulting equivalence relation  $R_E(X^C)$  and form a system of limitations

$|\eta_j(\lambda)| \leq \lambda_{N+1}, \quad j = \overline{1, n_E}$  as:

$$\left\{ \begin{array}{l} -\eta_j(\lambda) + \lambda_{N+1} \geq 0; \\ \eta_j(\lambda) + \lambda_{N+1} \geq 0, \quad j = \overline{1, n_E}; \\ \eta_{n_E+1}(\lambda) \equiv \sum_{i=1}^N \lambda_i = 1, \quad \lambda_i \geq 0, \quad i = \overline{1, N}. \end{array} \right. \quad (25)$$

Minimization  $\lambda_{N+1} \rightarrow \min$  within the constraints (25) is a linear programming problem, and provides a Chebyshev point of the system (22). Geometrically

Chebyshev point  $\lambda^o$  in this case has the smallest deviation in absolute value  $|r|$  from the entire system of planes described by the system of equations (22):

$$|r| = \min_{\lambda} \max_j |\eta_j(\lambda)| = \max_j |\eta_j(\lambda^o)|. \quad (26)$$

Let's introduce an additional variable  $\lambda_{N+1}$  in the constraints (23) for the relation  $R_S(X^C)$  and we require that the conditions  $\eta_j(\lambda) \leq \lambda_{N+1}, \quad j = \overline{1, n_S}$ . Then the search for the Chebyshev point of the system of inequalities (23) reduces to the problem of linear programming:

$$\left\{ \begin{array}{l} \lambda_{N+1} \rightarrow \min; \\ -\eta_j(\lambda) + \lambda_{N+1} \geq 0, \quad j = \overline{1, n_S}; \\ \eta_{n_S+1}(\lambda) \equiv \sum_{i=1}^N \lambda_i = 1, \quad \lambda_i \geq 0, \quad i = \overline{1, N}. \end{array} \right. \quad (27)$$

If the system of inequalities (23) is consistent, then  $r = \min_{\lambda} \max_j \eta_j(\lambda) \leq 0$  and the obtained solution  $\lambda^o$  will be maximally stable to possible displacements of the planes of constraints. If the system (23) is inconsistent, then  $r > 0$ , and we obtain the Chebyshev approximation, which is the value of the minimal deviation for the solution of the system under consideration. In this case, for a preference system described by a binary relation  $R_S(X^C)$ , there is no single weight vector of partial criteria  $\lambda$ , satisfying conditions (23).

Similarly, the problem of linear programming reduces to finding a Chebyshev solution (approximation) of a system of linear inequalities and constraints for the ratio of nonstrict preferences  $R_N(X^C)$  (24).

The disadvantage of solutions in the form of a Chebyshev point is their orientation solely on extreme constraints and minimizing the maximum deviation of the obtained point from the planes of constraints  $\eta(\lambda)$ . As an alternative to solutions in the form of a Chebyshev point one can use generalized solutions of systems (22) - (24), taking into account the removal (or evasion) of the entire set of constraints [16]. In this case, for the equivalence relation  $R_E(X^C)$  as the solution of the system of equations (22) is the vector:

$$\lambda^o = \arg \min_{\lambda} \|A\lambda - b\|, \quad (28)$$

where:  $\|A\lambda - b\|$  – the vector of the discrepancy vector;  $A = [a_{ij}]$  – matrix of coefficients for the system (22), whose elements are:

$$a_{ji} = [\xi_i(y) - \xi_i(x)], \quad j = \overline{1, n_E}, \quad i = \overline{1, m},$$

where:  $j$  – pair number  $\langle x, y \rangle$  with reference to  $R_E(X^C)$ ;  $a_{n_E+1, i} = 1, \quad i = \overline{1, m}$ ;  $b = [0, 0, \dots, 1]^T$ .

The proposed models and methods showed their efficiency and effectiveness in solving problems of design and management of large-scale objects.

### CONCLUSIONS

Within the framework of solving the problem of identification of preferences of DM in decision support systems, an analysis of existing models of multicriteria estimation was carried out. It is established that the known utility functions that make it possible to realize S- and Z-shaped dependencies on the values of partial criteria have a high computational complexity and rapidly change their values as the characteristics approach extrema. In practice, this can lead to significant errors in determining the properties of the variants for individual indicators and, as a consequence, to the error of their complex multicriteria evaluation.

To overcome these shortcomings, a modification of the gluing function and an efficient procedure for calculating its values are proposed, which allow reducing the dead zone substantially without loss of accuracy, thereby increasing the adequacy of the multifactor estimation model and shortening the calculation time of its values.

For the parametric synthesis of universal multicriterion estimation models based on the Kolmogorov-Gabor polynomial, an improvement in the method of comparator identification by calculating the Chebyshev point and the discrepancy vector is proposed. This allows to cover all practically important situations of choosing by DM, described by binary relations of equivalence, strict, nonstrict preferences and to increase the efficiency of synthesis procedures in comparison with the GMDH based on genetic algorithms.

Practical use of the obtained results in DSS of design and management decisions will allow to obtain solutions of problems of multifactor estimation and choice of solutions of much larger dimension with less expenses of computing resources practically without loss of accuracy.

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