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ABOUT ONE CLASS OF THE PROBLEMS OF OPTIMAL STOCHASTIC CONTROL OF HYBRID DYNAMICAL SYSTEMS

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Abstract. A new class of the problems of optimal stochastic control of hybrid dynamical systems different from well-known ones by the introduction of additional extreme and probabilistic constraints on the phase variables is studied in the present work. The mathematical formulation and approximate method of solution of the examined class of the problems are presented in this work. The effectiveness of the use of this class of the problems is illustrated on the example of one of the largest water main of Ukraine.

Key words: optimal stochastic control, hybrid dynamical systems, probabilistic constraints on the phase variables, water main.

INTRODUCTION

Pipeline Energy Systems (PES) play an important role in the energetics, industry, communal household sector and include oil and gas pipelines, water supply systems [1].

A special feature of PES is the presence in them some specific technological elements: underground gas storages, tank batteries, clean water reservoirs (CWR), receiving tanks. The dynamic properties of the processes of changes of the levels of target products in reservoirs or overpressure in the underground gas storages differ significantly from the dynamic properties of the transport processes of the target products on the sections of the pipeline.

If the dynamic processes in the technological elements of the system can be divided into two disjoint classes - fast and slow flowing and the dynamic properties of the system are determined by the dynamic properties of the interaction of the end system of the technological elements with slowly flowing processes, then such a system will be called a hybrid dynamical system. This class of the systems includes pipeline energy system, involving main pipelines of the target products (water, oil and oil products, condensation and so on) with tank batteries both at its inputs and outputs and within the system, multi crossing line sections and multi shop pumping stations (PS).

As an object of control hybrid dynamical systems belong to the class of multi-dimensional, multi connected, nonlinear dynamical systems.

In this work hybrid dynamical systems are considered as a stochastic object functioning in a stochastic environment. The stochastic properties of the object of control are manifested in the fact that the parameters of technological equipment are unknown authentically a priori, but replaced by the estimates obtained from experimental samples of the finite length, which are random variables.

The stochastic properties of the environment appear as main disturbing factors – processes of supply and consumption of the target products, which are random processes, dependent on three main groups of the factors - meteorological, chronological, organizational [2].

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

The whole technological process of transportation and distribution of the target products in PES is based on the use of the energy developed by pumping stations, which, in their turn, are major consumers of the electricity. In the cost price of transportation and distribution of the target product in PES the electricity cost reaches more than 90% [3-5]. The continuous increase of the electricity tariff taking place now in Ukraine has led to the urgent need in the development and implementation of new energy and resource saving technologies of the control in PES [2]. The analysis of the actual modes of operation of PES [6-9] has shown that in modern PES there are huge reserves of saving of material and energy costs, but their implementation requires the development of new mathematical models and methods of control of the technological process of production, preparation, transportation and distribution of the target products of PES. One of the main way of the saving of material and energy resources is the transition from the traditional to the optimal control with the use of an adjustable valve for pumping units (PU) [10-14]. The practical implementation of three-band tariffs for the electricity opens up new opportunities of the saving of financial costs for the electricity due to more efficient use of storage capacity of the tank batteries of the main pipelines [15-20].

In this work we introduce a new class of the problems of optimal stochastic control of hybrid dynamical systems with three-band tariff for the electricity for the practical implementation of these two possibilities.

OBJECTIVES

The purpose of the study is the selection of a new class of the problems of optimal stochastic control of hybrid dynamical systems.

To achieve this goal the following problems are being solved:

1. The mathematical formulation of a new class of the problems of optimal stochastic control of hybrid dynamical systems with a three-band tariff for the electricity.
2. The evaluation of the potential of resource and energy saving while the implementation of the examined problem in the systems of operational dispatch management of the modes of operation of the water main.

MAIN RESULTS OF THE RESEARCH

For the mathematical formulation of this class of the problems the following must be known: the structure of the system presented as an oriented graph the inputs and outputs of which are reservoirs; the parameters of multiline pipelines, multi shop pumping stations and reservoirs. The interval of adjustment [0, *T*] (one day), which is divided into 24 sub-intervals corresponding to each hour of the period of control $k = 0, \dots, 23$. At each k time sub-interval the predictions of inflow of the target products into the reservoirs at the inputs of the system, the incidental selections and the selections from the reservoirs at the output of the system in the form of conditional mathematical expectations

$$\bar{q}_{i0}(l) = \underset{\omega}{M}(q_{il}(\omega))$$

and their dispersions

$$\sigma^2_{q_{i0}}(l) = \underset{\omega}{D}(q_{il}(\omega))$$

calculated at the time interval $k = 0$ proactively $l = 1, 2, \dots, 23$; the measured values of the levels of the target products in each z reservoir $H_{zk}(\tilde{\omega})$; the actual quantity of the switched PU are known.

The objective function of the problem of optimal stochastic control of the modes of operation of hybrid dynamical systems is presented in the form of the mathematical expectation of the integrated cost of the electricity consumed by all operating PU at the interval of control [0, T]:

$$\underset{\omega}{M}\sum_{k=0}^{23}\sum_{i=1}^{n}\sum_{j=1}^{m_i} N_{ijk}(q_{ik}(\omega))\cdot r_k \to \min_{u(k)\in\Omega}, \qquad (1)$$

where: $N_{ijk}(q_{ik}(\omega))$ – the power expended by j PU of i PS at the k time interval; r_k – the value of three-band tariff for the electricity at k time interval; n – the number of PS; m_i – the number of PU operating on i PS.

The area of constraints Ω is determined by the stochastic model of quasi-stationary modes of transportation and distribution of the target products in the pipeline energy systems [2, 21]:

$$\underset{\omega}{\mathrm{M}}\Big(H^{\alpha}_{r\text{н}}(\omega) - H^{\alpha}_{r\text{к}}(\omega) + \sum_{i\in L} b_{1ri} h_{NAik}(q_{ik}(\omega)) + \sum_{i\in R} b_{1ri} h_{RZik}(q_{ik}(\omega)) + \sum_{i\in M_1} b_{1ri}(H^{\alpha}_{i\text{н}}(\omega) - H^{\alpha}_{i\text{к}}(\omega))\Big) = 0,$$

$$(r = v, \dots, v+\eta_2-1; \quad k = 0, \dots, 23), \qquad (2)$$

$$\underset{\omega}{\mathrm{M}}\Big(h^{c}_{r}(q_{rk}(\omega)) - H_{1k}(\omega) + \sum_{i\in L} b_{1ri} h_{NAi}(q_{ik}(\omega)) + \sum_{i\in R} b_{1ri} h_{RZi}(q_{ik}(\omega)) + \sum_{i\in M_1} b_{1ri}(H^{\alpha}_{i\text{н}}(\omega) - H^{\alpha}_{i\text{к}}(\omega) + h^{g}_{i})\Big) = 0,$$

$$(r = v+\eta_2, \dots, v+\eta_2+\xi_1-1), \qquad (3)$$

$$\underset{\omega}{\mathrm{M}}\Big(H^{\alpha}_{r\text{н}}(\omega) - H^{\alpha}_{r\text{к}}(\omega) + h^{g}_{r} - H_{zk}(\omega) + H_{1k}(\omega) + \sum_{i\in L} b_{1ri} h_{NAi}(q_{ik}(\omega)) + \sum_{i\in R} b_{1ri} h_{RZi}(q_{ik}(\omega)) + \sum_{i\in M_1} b_{1ri}(H^{\alpha}_{i\text{н}}(\omega) - H^{\alpha}_{i\text{к}}(\omega) + h^{g}_{i})\Big) = 0,$$

$$(r = v+\eta_2+\xi_1, \dots, e; \quad z = 1, \dots, Z), \qquad (4)$$

$$\underset{\omega}{\mathrm{M}}\left(\sum_{r=v}^{v+\eta_2-1} b_{1ri} q_{rk}(\omega) + \sum_{r=v+\eta_2}^{e} b_{1ri} q_{rk}(\omega) - q_{ik}(\omega)\right) = 0,$$

$$\big(i = 1, \dots, v-1\big), \qquad (5)$$

The temperature of the target product at any point x of the pipeline section is found as follows:

$$T_x = T_{cp} + (T_{\text{н}} - T_{cp})e^{-\theta x}, \qquad (6)$$

where: x – the distance from the beginning of the section to the point of this section with the coordinate *x*; T_{cp} – the average at a certain time interval temperature of the environment where the section of the pipeline is situated; $T_{\text{н}}$ – the temperature of the target product at the beginning of the section; θ – heat transfer parameter [2].

If in j nod there is some mixing m of the incoming streams of the target product with different temperature $T_i(\omega), \quad (i = 1, 2, \dots, m)$, the temperature of the target product $T_j(\omega)$, which outputs out of this nod is determined as follows:

$$\underset{\omega}{\mathrm{M}}\left(\sum_{i=1}^{m} T_{ik}(\omega)q_{ik}(\omega) - T_{jk}(\omega)\sum_{i=1}^{m} q_{ik}(\omega)\right) = 0,$$
$$\left(j = 1,\ldots,v\right), \tag{7}$$

$$q_{ik}\left(\omega\right) > 0, \quad i \in L. \tag{8}$$

$$H^{\alpha}_{i\text{н}}(\omega) - H^{\alpha}_{i\text{к}}(\omega) = sgn\, q_{ik}(\omega) S_i(\omega) q^2_{ik}(\omega),$$
$$i \in M, \quad \alpha \geq 1, \tag{9}$$

$$h_{NAik}(q_{ik}(\omega)) = a_{0i}(\omega) + a_{1i}(\omega)q_{ik}(\omega) +$$
$$+ a_{2i}(\omega)q^2_{ik}(\omega), \quad i \in L, \tag{10}$$

$$\eta_{NAik}(q_{ik}(\omega)) = d_{0i}(\omega) + d_{1i}(\omega)q_{ik}(\omega) +$$
$$+ d_{2i}(\omega){q_{ik}}^2(\omega), \quad i \in L, \tag{11}$$

$$N_{NAik}(q_{ik}(\omega)) = \frac{9{,}81 \cdot h_{NAik}(q_{ik}(\omega)) \cdot q_{ik}(\omega)}{0{,}9 \cdot \eta_{NAik}(q_{ik}(\omega))},$$
$$i \in L, \tag{12}$$

$$h_{RZik}(\mathrm{q}_{ik}(\omega)) \;=\; \frac{\mathrm{q}_{ik}(\omega)\mathrm{C}_i(\omega)}{\mathrm{E}^2_{ik}}, \quad \mathrm{i} \in \mathrm{R}, \tag{13}$$

and dynamical models of the reservoirs

$$H_{zk}(\omega) = H_{zk-1}(\omega) + c_{zk}(q_{zvhk}(\omega) - q_{zvihk}(\omega)),$$
$$(z = 1,\ldots,Z), \tag{14}$$

with probabilistic constraints on the phase variables:

$$P\left(H_{zk}(\omega) \leq H^{\max}_{z}\right) \geq \alpha,$$
$$P\left(H_{zk}(\omega) \geq H^{\min}_{z}\right) \geq \alpha, \quad \alpha \approx 0{,}97, \tag{15}$$

and extreme values of the constraints on the phase variables for the settled time interval $k = 6$ and $k = 23$:

$$\underset{\omega}{M}\{H_{z6}(\omega)\} \to \underset{q_{zvhk}\in\Omega}{extr}, \quad (k = 0,\ldots,6), \tag{16}$$

$$\underset{\omega}{M}\{H_{z23}(\omega)\} \to \underset{q_{zvhk}\in\Omega}{extr}, \quad (k = 0,\ldots,23), \tag{17}$$

where: $u(k)$ – vector of control which determines the amount of operating PU, the position of adjustable valves (AV); $H_{zk}(\omega)$ – level of the target product in z reservoir at a given k time interval, $H^{\min}_{z}$, $H^{\max}_{z}$ – minimum and maximum allowable level of the target product in each z reservoir.

Random variables characterize: $q_{ik}(\omega)$ – consumption of the target product on i section of the pipeline at k time interval; $H^{\alpha}_{i\text{н}}(\omega) - H^{\alpha}_{i\text{к}}(\omega)$ – pressure drop on i section of the pipeline; $h_{NAik}(q_{ik}(\omega))$ – pressure of i PU; $S_i(\omega) = f(l_i, d_i, T_{cpi}, k_i)$ – hydraulic resistance of i section of the pipeline ($i \in M$), k_i – coefficient of heat transfer from the target product to the environment; $h_{RZik}(\mathrm{q}_{ik}(\omega))$ – evaluation of the pressure drop on i AV; $\eta_{NAik}(q_{ik}(\omega))$ – evaluation of the coefficient of the efficiency of i PU; $a_{0i}(\omega), a_{1i}(\omega), a_{2i}(\omega), d_{0i}(\omega), d_{1i}(\omega), d_{2i}(\omega)$ – evaluation of the parameters of PU ($i \in L$); $C_i(\omega)$ – evaluation of the parameters of AV ($\mathrm{i} \in \mathrm{R}$); E_{ik} – opening degree of AV (E (0,1]); l_i, d_i, h^g_i – length, diameter and geodesic mark of i section of the pipeline ($i \in M$), b_{1ri} – cyclomatic matrix element; $q_{zvh}(\omega)$, $q_{zvih}(\omega)$ – consumption of the target product at input and output of the reservoir; $\underset{\omega}{M}\{\cdot\}$ – mathematical expectation of the random variable {.}.

For the solvability of the problem (1–17), the system of equations (2–17) is supplemented with the boundary conditions in the form of the predictions of the consumption of all consumers of the system $q_{ik}(l)$ (incidental and end) at all inputs, the predictions of the temperature of the target product supplied into the inputs of the main pipeline, calculated as conditional mathematical expectations at the time interval $k = 0$, proactively $l = 1,2, \ldots, 23$ as well as the initial conditions when $k = 0$ is in the form of estimates of the mathematical expectations of the levels of the target product in each z reservoir – H_{z0}.

Extreme values of the constraints on the phase variables are determined by the specifics of the object of control and the three-band tariff for the electricity. For the main pipelines in the restriction (16) *extr* is replaced by *min*, in the restriction (17) *extr* is replaced by *max* [3, 4]. For the sewerage systems in the restriction (16) *extr* is replaced by *max*, in the restriction (17) *extr* is replaced by *min* [5]. In the work [3] an approximate method of solving the problem in question by the transition from a stochastic problem (1 – 17) to its deterministic equivalent is presented here.

The deterministic equivalent of the problem of optimal stochastic control of the modes of operation of hybrid dynamical systems at the time interval [0, *T*] takes the form:

$$\sum_{k=0}^{23}\sum_{i=1}^{n}\sum_{j=1}^{m_i} \bar{N}_{ijk}(\bar{q}_{ik}) \cdot r_k \to \min_{u(k)\in\Omega}, \tag{18}$$

$$\bar{H}^{\alpha}_{r\text{н}} - \bar{H}^{\alpha}_{r\text{к}} + \sum_{i\in L} b_{1ri}\bar{h}_{NAik}\;(\bar{q}_{ik}) +$$
$$+ \sum_{i\in R} b_{1ri}\bar{h}_{RZik}\;(\bar{q}_{ik}) + \sum_{i\in M_1} b_{1ri}(\bar{H}^{\alpha}_{i\text{н}} - \bar{H}^{\alpha}_{i\text{к}}) = 0,$$
$$(r = v,\ldots,v+\eta_2-1; \quad k = 0,\ldots,23), \tag{19}$$

$$\bar{h}_r^{c}(\bar{q}_{rk}) - \bar{H}_{1k} + \sum_{i\in L} b_{1ri}\bar{h}_{NAi}(\bar{q}_{ik}) + \\ + \sum_{i\in R} b_{1ri}\bar{h}_{RZi}(\bar{q}_{ik}) + \sum_{i\in M_1} b_{1ri}\big(\bar{H}^{\alpha}_{i\text{н}} - \bar{H}^{\alpha}_{i\text{к}} + h_i^{g})\big) = 0, \\ (r = v+\eta_2,\dots,v+\eta_2+\xi_1-1), \quad (20)$$

$$\bar{H}^{\alpha}_{r\text{н}} - \bar{H}^{\alpha}_{r\text{к}} + h_r^{g} - \bar{H}_{zk} + \bar{H}_{1k} + \\ + \sum_{i\in L} b_{1ri}\bar{h}_{NAi}(\bar{q}_{ik}) + \sum_{i\in R} b_{1ri}\bar{h}_{RZi}(\bar{q}_{ik}) + \\ + \sum_{i\in M_1} b_{1ri}\big(\bar{H}^{\alpha}_{i\text{н}} - \bar{H}^{\alpha}_{i\text{к}} + h_i^{g})\big) = 0, \\ (r = v+\eta_2+\xi_1,\dots,e; \quad z = 1,\dots,Z), \quad (21)$$

$$\bar{q}_{ik} = \sum_{r=v}^{v+\eta_2-1} b_{1ri}\bar{q}_{rk} + \sum_{r=v+\eta_2}^{e} b_{1ri}\bar{q}_{rk}, \\ \big(i = 1,\dots,v-1\big). \quad (22)$$

$$\bar{q}_{ik} > 0, \quad i \in L. \quad (23)$$

$$\bar{H}^{\alpha}_{i\text{н}} - \bar{H}^{\alpha}_{i\text{к}} = sgn\,\bar{q}_{ik}\bar{S}_i\bar{q}_{ik}^{2}, \\ i \in M, \quad \alpha \ge 1 \quad (24)$$

$$\bar{h}_{NAik}(\bar{q}_{ik}) = \bar{a}_{0i} + \bar{a}_{1i}\bar{q}_{ik} + \bar{a}_{2i}\bar{q}_{ik}^{2}, \quad i \in L, \quad (25)$$

$$\bar{\eta}_{NAik}(\bar{q}_{ik}) = \bar{d}_{0i} + \bar{d}_{1i}\bar{q}_{ik} + \bar{d}_{2i}\bar{q}_{ik}{}^{2}, \quad i \in L, \quad (26)$$

$$\bar{N}_{NAik}(\bar{q}_{ik}) = \frac{9{,}81\cdot\bar{h}_{NAik}(\bar{q}_{ik})\cdot\bar{q}_{ik}}{0{,}9\cdot\bar{\eta}_{NAik}(\bar{q}_{ik})}, \\ i \in L, \quad (27)$$

$$\bar{h}_{RZik}(\mathrm{q}_{ik}) = \frac{\bar{\mathrm{q}}_{ik}\bar{\mathrm{C}}_i}{\mathrm{E}_{ik}^{2}}, \quad \mathrm{i} \in \mathrm{R}, \quad (28)$$

$$\bar{H}_{zk} = \bar{H}_{zk-1} + c_{zk}(\bar{q}_{zvhk} - \bar{q}_{zvihk}), \\ (z = 1,\dots,Z). \quad (29)$$

$$\bar{H}_{zk} \le H_z^{\max}, \quad \bar{H}_{zk} \ge H_z^{\min}, \quad (30)$$

$$H_{z6} \to \underset{q_{zvhk}\in\Omega}{extr}, \ (\mathrm{k} = 0,\dots,6), \quad (31)$$

$$H_{z23} \to \underset{q_{zvhk}\in\Omega}{extr}, \ (\mathrm{k} = 0,\dots,23). \quad (32)$$

The problem (18–32) belongs to the class of the problems of nonlinear dynamic programming with nonlinear objective function (18), non-linear constraints in the form of qualities (19–21), extreme constraints (31–32), coupling equations (22–29) and the unilateral restrictions variables (23), (30).

The solution of the problem (18–32) is carried out by a modified method of branches and borders [3].

THE EVALUATION OF THE POTENTIAL OF RESOURCE AND ENERGY SAVING OF THE WATER MAIN

Practical use of the examined class of the problems in the systems of the operational dispatch management allows to embody not only energy and resource saving technologies, but also to get unbiased, efficient and consistent evaluations of the potential of resource saving while the modernization of technological equipment of WM, which is the basis for the investment projects. Without loss of generality, we will assess the potential of energy and resource saving for WM at the beginning while the transition from the traditional system of control of the modes of operation of WM to the system of optimal stochastic control with the following possible variants of modernization of technological equipment. The evaluation of the potential of resource and energy saving was carried out for one of the largest Ukrainian WM the scheme of which is shown in Fig. 1.

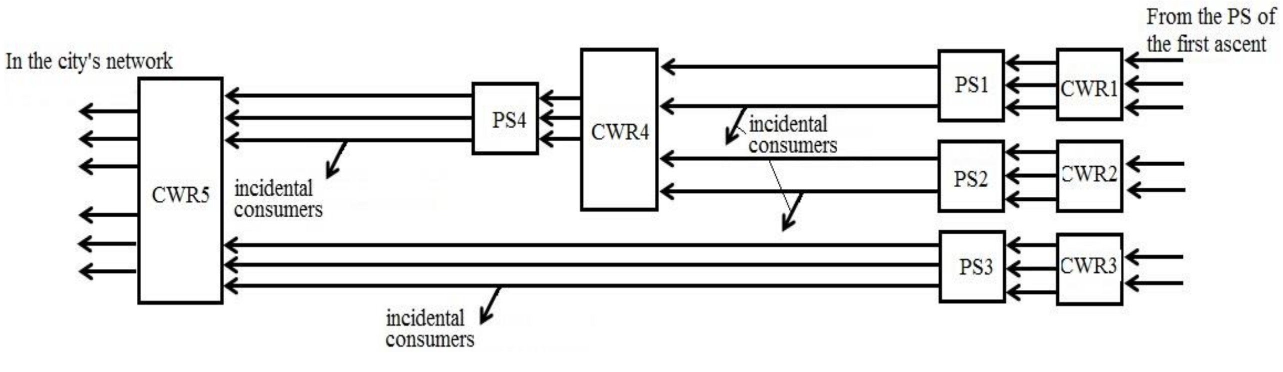

Fig. 1. The structure of the water main

The structuring of the examined class of the problems applied to WM is given in the work [3]. The use of the system of optimal stochastic control has initially allowed to evaluate the potential of resource saving while the transition from the existing system of operational dispatch management of the modes of operation of WM to the system of optimal stochastic control with three-band tariff for the electricity. Further the evaluation of the potential of energy and resource saving, associated with the modernization of technological equipment, was carried out for the system of optimal stochastic control.

The energy cost is determined according to three-band tariff (see Table 1).

Table 1. The electricity rate according to the hours

Hours	Coefficient	Cost of 1 кW/h, UAH
6.00–8.00	1,02	1,6
8.00–10.00	1,8	2,82
10.00–18.00	1,02	1,6
18.00–22.00	1,8	2,82
22.00–23.00	1,02	1,6
23.00–6.00	0,35	0,39

The examined WM includes PS of the second lift (PS1, PS2, PS3) and PS4 of the third lift. There are clean water reservoirs (CWR) at the input of each PS and at the output of WM.

The input data for the problem of optimal stochastic control of the modes of operation of WM at the time interval $[0, T]$ (7 days) are as follows:

- static data including the structure of WM: lengths; diameters; geodetic marks of the sections of the pipeline; the estimates of the parameters of the mathematical models of PU for each PS; the estimates of hydraulic resistances of AV on each PS; physical dimensions of each of CWR;
- dynamic data, including the prediction of the daily water consumption from CWR5; the prediction of the water consumption by incidental consumers.

PS of the second lift PS1, PS2 and PS3 are equipped with the same type, connected in parallel PU with the same characteristics, PS4 of the third lift is equipped with the same type PU with different characteristics. At time zero k=0 the mathematical expectation of the water levels in CWR4 is $H_{1,0} = 2,6$ m; in CWR5 $H_{2,0} = 3,9$ m. The allowable ranges of change in water levels in CWR4 is [2-4.9], in CWR5 is [1.45-4.9]. The actual parameters of WM the technological equipment are given in Table 2.

Table 2. The actual parameters of the process equipment of the MW

PS	PU type	q, m^3/h	h, m	N, kW	n_1, min^{-1}.	d, mm	amount of PU
PS1	20NDS	3420	71	960	1000	765	4
PS2	24NDS	6500	79	1600	750	1040	5
PS3	22NDS	4799	90	1000	1250	825	4
PS4	24NDS	6500	79	1600	750	990–1040	6

Table 3 shows the parameters of the modes of operation of WM for the actual operating mode and the operating mode at the optimal stochastic control (S – costs for the electricity).

Table 3. The estimates of the parameters of the modes of operation of WM at the time interval 7 days

Day	Actual modes		Optimal modes	
	N, kW	S, UAH	N, kW	S, UAH
1	261567	390840	288533	362761
2	265399	396530	300658	386446
3	258970	393765	293739	370907
4	251876	391068	289142	363551
5	251998	391339	281864	352021
6	269445	398191	298675	385346
7	270893	400120	279714	361951
Amount	1830152	2761856	2032327	2582988

From Table 3 it can be seen that the transition from the existing system of control of the modes of operation of WM to the system of optimal stochastic control allowed to reduce the actual costs for the electricity by 6.5%.

Table 4 shows 6 variants of modernization of the technological equipment of WM for the existing structure of WM and 2 variants for WM with the altered structure. The point of the structural changes of WM was in switching of the outputs of PS3 to the inputs of CWR4 in accordance with the scheme shown in Fig. 2.

Fig. 3, Fig. 4 show the estimates of the mathematical expectation of the power and energy costs and the potential of resource and energy saving at the time interval 7 days for the existing system of control (f), for the system of optimal stochastic control (s) and eight different variants of modernization.

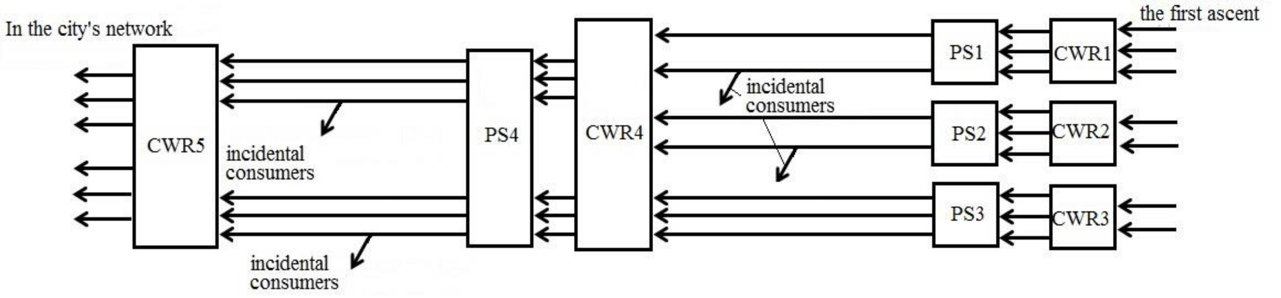

Fig. 2. The scheme a modified water main structure

Table 4. The variants of modernization of the technological equipment of WM

Variant	The point of modernization	Actual parameters of the operating equipment	Changed parameters of the operating equipment
1	Replacing the engine to the PS1	$n1$=960 min^{-1}	$n1^*$=750 min^{-1}
2	Cutting wheels of PU on PS1	d=765 mm	d^*=612 mm
3	Cutting wheels of PU on PS2	d=990 mm	d^*=792 mm
4	Replacing the engine to the PS1, cutting wheels of PU on PS2	$n1$=960 min^{-1} d=990 mm	$n1^*$=750 min^{-1} d^*=792 mm
5	Cutting wheels of PU on PS1, cutting wheels of PU on PS2	d=765 mm d=990 mm	d^*=612 mm d^*=792 mm
6	Cutting wheels of PU on PS1 cutting wheels of PU on PS2, one adjustable drive on PS3, two adjustable drives on PS4.	d=765 mm d=990 mm $n1$=1000 min^{-1} $n1$=750 min^{-1}	d^*=612 mm d^*=792 mm $n1^*$=700 min^{-1} $n1^*$=650 min^{-1}
7	Modification of the structure of WM, cutting wheels of PU on PS1, cutting wheels of PU on PS2, cutting wheels of PU on PS3, cutting wheels of PU on PS4,	d=765 mm d=990 mm d=825 mm d=990 mm	d^*=612 mm d^*=792 mm d^*=660 mm d^*=792 mm
8	Modification of the structure of WM, cutting wheels of PU on PS1, cutting wheels of PU on PS2, cutting wheels of PU on PS3, two adjustable drives on PS4,	d=765 mm d=990 mm d=825 mm $n1$=750 min^{-1}	d^*=612 mm d^*=792 mm d^*=660 mm $n1^*$=650 min^{-1}

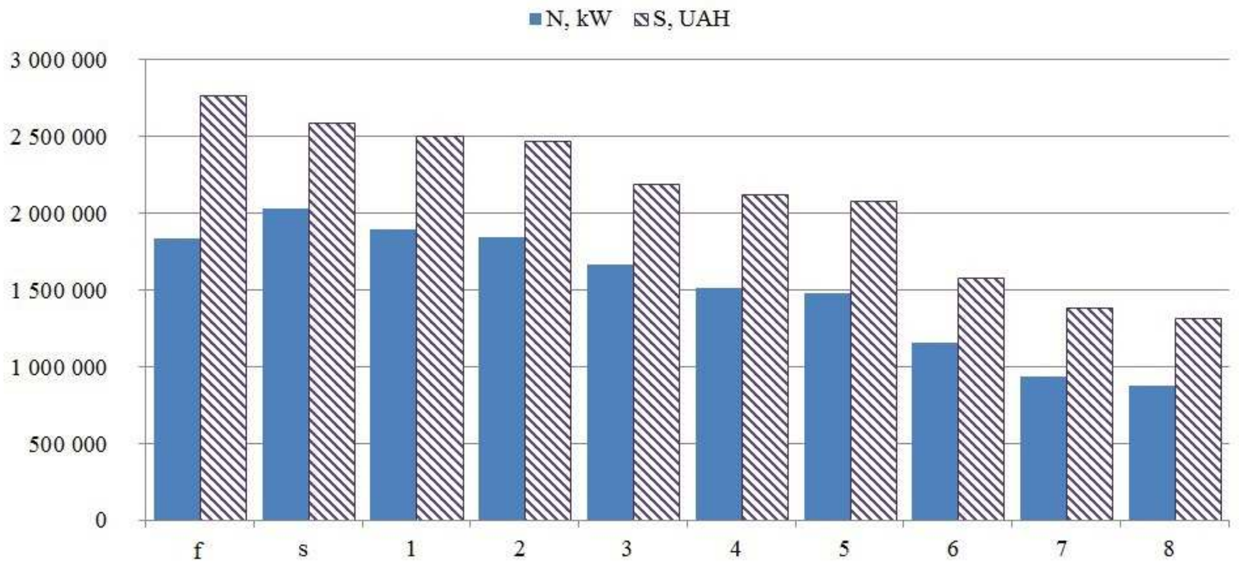

Fig. 3. The estimates of the mathematical expectation of the power and energy costs at the time interval 7 days for the existing system of control (f), for the system of optimal stochastic control (s) and eight different variants of modernization WM

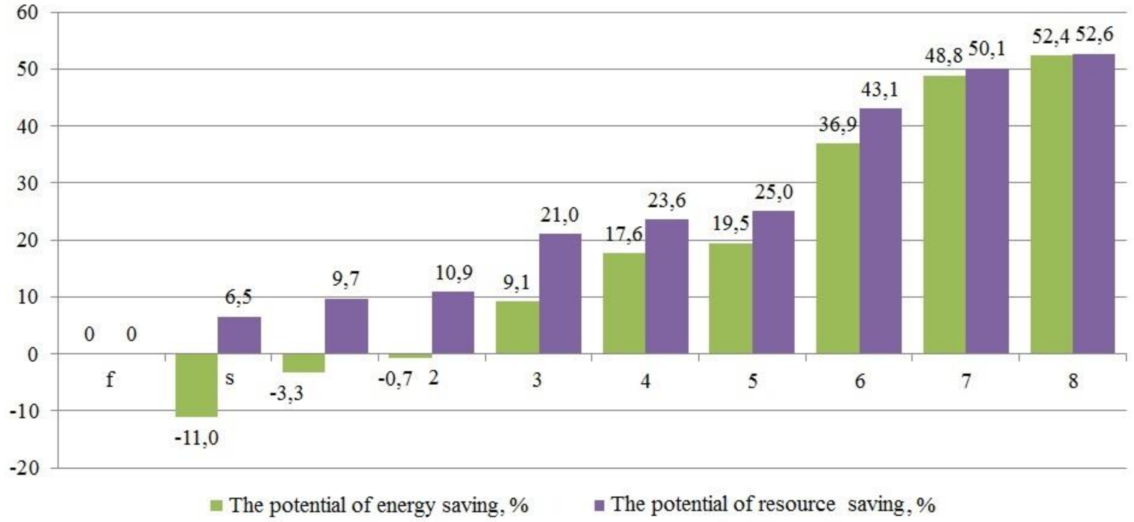

Fig. 4. The estimates of the mathematical expectation of the potential of resource and energy saving at the time interval 7 days for the existing system of control (f), for the system of optimal stochastic control (s) and eight different variants of modernization WM

The analysis of the obtained results showed that all variants of modernization have significant potential of resource and energy saving, but for its implementation require various material costs. Among the least expensive variants of modernization connected with cutting wheels on PU, variant 5 with the potential of energy saving 19.5% and resource saving 25% must be emphasized. More promising, with the potential of energy saving 52.4% and resource saving 52.6% is variant 8 providing cutting wheels of PU and installation of two adjustable valves on PS4. It should be noted that all examined variants of the modernization of WM are economically attractive, and the payback period does not exceed one year.

CONCLUSIONS

In the present work:

1. A new class of the problems of optimal stochastic control of hybrid dynamical systems different from existing ones by the introduction of additional extreme and probabilistic constraints on the phase variables, the use of which allows to implement practically the energy saving technologies in the PES is proposed;
2. It is shown that the use of this class of the problems allowed to obtain unbiased, efficient and consistent estimates of the potential of resource and energy saving for one of the largest water main of Ukraine.

The obtained results are the basis for the development of science-based investment projects on the modernization of WM.

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