

# Research of the Flow-based Model of Hierarchical Multicast Routing

Olena Nevzorova, Kinan M. Arous, Ali Salem Ali

**Abstract** - To improve scalability of multicast routing solutions the method of hierarchical routing with coordination was proposed. It is based on the decomposition representation of the flow-based model of multicast routing. The usage of this method allowed decentralizing the calculation of the multicast routes in the border routers of telecommunication networks.

**Keywords** - Hierarchical Routing, Multicast, Flow-based Model, Convergence, Iteration.

## I. INTRODUCTION

The rapid development of telecommunications and the development new multimedia applications justifies the transition to NGN (New Generation Network). NGN is a universal multi-purpose network for transmitting different services traffic: video, voice, images, data, and others. Applications like IP-radio, IP-TV, video and audio conferencing attracted such forms of broadcasting as one-to-many and many-to-many. This type of broadcasting assumes necessity of realization flow-based multicast routing. Currently, the set of multicast routing protocols such as DVMRP (Distance Vector Multicast Routing Protocol), Multicast Расширения OSPF (MOSPF), PIM (Protocol Independent Multicast) is widely used in practice [1]. But very important problem is the increasing of scalability of multicast routing solutions with QoS (Quality of Service). The traditional way out of this situation is the transition to a hierarchical (multi-level) solution that combines the advantages of both centralized and decentralized (distributed) management. Therefore, the actual problem relates with development the method of hierarchical multicast routing with coordination.

## II. DECOMPOSED MODEL OF MULTICAST ROUTING

In order to develop the method of hierarchical routing with coordination the flow-based model (which developed in [2]) should be represented in decomposed form.

Let us describe a network structure as oriented graph  $G = (M, E)$ , where  $M = \{M_i, \overline{i, m}\}$  is a set of vertices – nodes (routers) of the network;  $E$  is a set of graph arcs

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modeling network links. For each link  $(i, j) \in E$  ( $(i, j)$ : link goes from  $i$  to  $j$ ), it is specified the bandwidth  $\varphi_{(i,j)}$  which measured in packet per second (1/s). Then, at the realization of multicast routing on source it is necessary to calculate the set of Booleans for every  $r$ -th router-sender:

$$x_{(i,j)}^{k_r} \in \{0;1\}, \quad (1)$$

which represent the share of  $k_r$ -th flow rate transmitted through  $(i, j) \in E$  link;  $k_r$  is flow entering to network from  $r$ -th border node ( $k_r \in K$ ),  $K$  is the set flows in network.

With each  $k_r$ -th flow the set of parameters was associated:

$\lambda^{k_r}$  is  $k_r$ -th flow rate (1/s);

$s_{k_r}$  is router-sender;

$$d_{k_r}^* = \{d_{k_r}^1, d_{k_r}^2, \dots, d_{k_r}^{m_{k_r}}\} \quad (2)$$

is set of router-receiver, where  $m_{k_r}$  is the number of receivers of  $k_r$ -th flow[2], [5].

Routing variables (1) are limited by several constraints:

$$\sum_{j:(i,j) \in E} x_{(i,j)}^{k_r} \geq 1 \quad \text{when } k_r \in K, M_i = s_{k_r}, \quad (3)$$

if  $i$ -th router is router sender and also

$$\sum_{i:(i,j) \in E} x_{(i,j)}^{k_r} = 1 \quad \text{when } k_r \in K, M_j \in d_{k_r}^*, \quad (4)$$

for every router-receivers.

Constraint (3) is introduced to the router sender and its implementation directs that flow from that node (incoming to service) will be transmitted to one adjacent node at least. Condition (4) aim to provide packet flow delivery to each destination node and flow should arrive to the node only from one adjacent node.

For every  $j$ -th transit node which can be any router, except the router-sender, is given the following conditions:

$$\sum_{i:(i,j) \in E} x_{(i,j)}^{k_r} \geq x_{(j,p)}^{k_r}, \quad (5)$$

when  $k_r \in K$ ,  $M_j \neq s_{k_r}$  and  $M_j \notin d_{k_r}^*$ .

The fulfillment of these conditions allows to have a flow in any link  $((j,p) \in E)$  coming from the transit

node only in that case when this flow comes on the given node at least via one incoming link  $((i, j) \in E)$ .

In order to prevent cycle forming the following conditions are entered in proposed model:

$$\sum_{(i,j) \in E_\pi^i} x_{(i,j)}^{k_r} < |E_\pi^i|, \quad (6)$$

where  $E_\pi^i$  is a set of arcs forming  $i$ -th cycle ( $\pi$ ) according to their orientation;  $|E_\pi^i|$  – denotes power of the set  $E_\pi^i$ . The fulfillment of the condition (6) guarantees that the number of arcs used in multicast routing, composing any cycle is always smaller than the total number of arcs in this cycle [2],[5].

In addition, to prevent the links overload it is important to fulfill the conditions:

$$\sum_{M_r \in M} \sum_{k_r \in K} \lambda^{k_r} x_{(i,j)}^{k_r} \leq \varphi_{(i,j)}, \quad (i, j) \in E. \quad (7)$$

Condition (7) oriented to the organization of centralized routing while all variables  $x_{(i,j)}^{k_r}$  ( $M_r \in M$ ,  $(i, j) \in E$ ,  $k_r \in K$ ) are defined on unified routes server. In implementing of hierarchical routing, (7) takes the following form [2]:

$$\sum_{k_r \in K} \lambda^{k_r} x_{(i,j)}^{k_r} \leq \varphi_{(i,j)} - \sum_{\substack{M_s \in M \\ s \neq r}} \sum_{k_s \in K} \lambda^{k_s} x_{(i,j)}^{k_s}. \quad (8)$$

The meaning of the condition (8) is that the rate of packets flow routed  $r$ -th border node mustn't exceed the available bandwidth, remaining after the service of the flows was received through other border routers.

The condition (8) can be represented in vector-matrix form

$$B_r \bar{x}_r \leq D_r \bar{\varphi} - \sum_{\substack{M_s \in M \\ s \neq r}} C_{rs} \bar{x}_s, \quad (9)$$

where  $\bar{x}_r$  is a vector which coordinates are variables  $x_{(i,j)}^{k_r}$

$$\bar{x}_r = \begin{bmatrix} x_{(1,2)}^1 \\ x_{(1,2)}^2 \\ \vdots \\ x_{(i,j)}^{k_r} \\ \vdots \\ x_{(m,m-1)}^{K_r} \end{bmatrix};$$

$\bar{\varphi}$  is a vector of links bandwidths with coordinates  $\varphi_{(i,j)}$

$$\bar{\varphi} = \begin{bmatrix} \varphi_{(1,2)} \\ \varphi_{(1,3)} \\ \vdots \\ \varphi_{(i,j)} \\ \vdots \\ \varphi_{(m,m-1)} \end{bmatrix};$$

$B_r$ ,  $D_r$ ,  $C_{rs}$  are matching matrices because the dimension of vectors  $\bar{x}_r$  ( $r \in M$ ) and  $\bar{\varphi}$ , and also the numbering of their coordinates in general may not coincide.

The objective is to minimize

$$\begin{aligned} & \min F, \\ & F = \sum_{M_r \in M} \bar{x}_r^t H_r \bar{x}_r, \end{aligned} \quad (10)$$

where  $H_r$  is diagonal matrix of weight coefficients which coordinates are generally metric of the links,  $[\cdot]^t$  is transpose function of the vector (matrix) [2].

### III. METHOD OF HIERARCHICAL MULTICAST ROUTING

To solve the optimization problem which was formulated (related to minimizing (10) with constraints (3)-(8)) we used goal coordination method [3]. Then turning to the problem of an unconditional extremum

$$\min_x F = \max_\mu L,$$

it is necessary to maximize the Lagrangian by  $\bar{\mu}$ :

$$\begin{aligned} L = & \sum_{M_r \in M} \bar{x}_r^t H_r \bar{x}_r + \\ & + \sum_{M_r \in M} \bar{\mu}_r^t (B_r \bar{x}_r - D_r \bar{\varphi} + \sum_{\substack{M_s \in M \\ s \neq r}} C_{rs} \bar{x}_s) \end{aligned} \quad (11)$$

where  $\bar{\mu}$  is vector of Lagrange multipliers, and  $\bar{\mu}_r$  – is  $\bar{\mu}$ -th subvectors related to each of the conditions (10).

In the framework of the goal coordination method lets represent the Lagrangian (11) in follow form:

$$L = \sum_{M_r \in M} L_r, \quad (12)$$

where

$$L_r = \bar{x}_r^t H_r \bar{x}_r + \bar{\mu}_r^t (B_r \bar{x}_r - D_r \bar{\varphi} + \sum_{\substack{M_s \in M \\ s \neq r}} \bar{\mu}_s^t C_{sr} \bar{x}_s), \quad (13)$$

supposing that the values  $\bar{\mu}_r$  and  $\bar{\mu}_s$  are fixed (if there are constraints (9)) and formed on upper level of hierarchy.

Thus objective (10) takes a separable form (12). The general routing problem was decomposed into a number of routing tasks (13). Optimization is realized by the two-level scheme [4],[5]. On the lower level there is a

calculation of routers (1) for multicast flows by router-senders. The main task of upper level is coordinating of the solutions obtained from lower level in order to prevent overloading of links (8) by modifying vector of Lagrange multipliers in performing a gradient iterative procedure:

$$\bar{\mu}_r(\alpha + 1) = \bar{\mu}_r(\alpha) + \nabla \bar{\mu}_r, \quad (14)$$

where  $\alpha$  is a number of iteration;  $\nabla \bar{\mu}_r$  is a gradient of the function (13) calculated from the results of calculation routing tasks  $\bar{x}_r^*$  ( $r \in M_r$ ) in each router-sender which was obtained at the upper level [4],[5]:

$$\nabla \bar{\mu}_r(x) \Big|_{x=x^*} = B_r \bar{x}_r^* - D_r \bar{\varphi} + \sum_{\substack{M_s \in M \\ s \neq r}} C_{rs} \bar{x}_s^*. \quad (15)$$

According to expressions (11)-(15) Fig. 1 shows the general scheme of method of hierarchical multicast routing with coordination in telecommunication network.

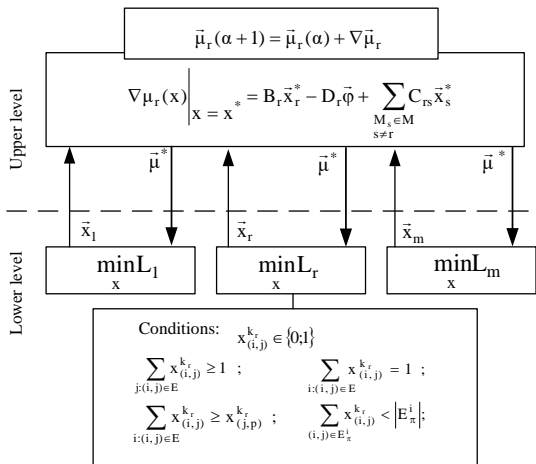


Fig. 1. General scheme of method of hierarchical multicast routing with coordination

#### IV. ANALYSIS OF METHOD OF HIERARCHICAL MULTICAST ROUTING

The effectiveness of the method (1)-(15) determines by the coordination procedure (14), (15) convergence rate. The less iteration is needed to obtain the desired optimum solution, the lower the volume of service traffic circulates in network. Thus, the process of the method (1)-(15) convergence was analyzed which objectives are, firstly, prove that method converges to the optimal solution, secondary, investigate the number of iterations of coordination procedure (14)-(15), and thirdly determine the degree of influence of the structural and functional parameters of the network and characteristics of circulating flows on the convergence of the procedure.

The research of the method of hierarchical multicast routing with coordination performed for networks structures with variable number of nodes (routers) and links. For example let us consider the network structure shown in Fig. 2.

The network consists of seven nodes (routers) ( $M_1 \div M_7$ ) and ten links with the bandwidth (packet per second, 1/s) shown on the graph arcs.

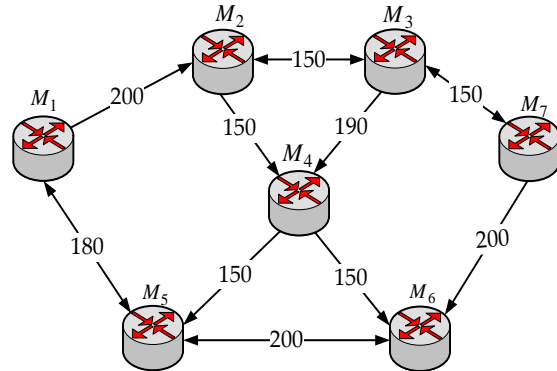


Fig. 2. The network structure was investigated

The number of flows equaled two, and their characteristics, acting as input data for calculation are presented in Table I.

TABLE I  
FLOWS CHARACTERISTICS

No of flow	Flow rate, 1/s	Router-sender	Routers-receivers
1	$\lambda^1 = 10 \div 50$	$M_1$	$M_3, M_4, M_5, M_6$
2	$\lambda^7 = 10 \div 50$	$M_7$	$M_2, M_4, M_5$
3	$\lambda^{21} = 10 \div 50$	$M_1$	$M_3, M_4, M_5, M_6$
4	$\lambda^{27} = 10 \div 50$	$M_7$	$M_2, M_4, M_5$

In this paper the dependence of number of iteration on flow rates was investigated. The investigation was conducted for a different number of flows circulating in the network. First of all let's consider the case when two flows circulate in the network. Flows rate was changed in step of 5 packets per second (1/s) from 10 to 100 1/s.

For two flows the analysis showed that if network load is less than 70-75% border routers calculate routes from sender to receiver without link overload and in this case coordination of their decision don't need. But with further increase of network load the distributed nature of the solutions obtained from each router overloaded links. To prevent link and network overload on the upper level of hierarchy the coordination of decisions obtained from lower level was occurred during calculation of Lagrange multipliers (14). Thus the coordination process has become an iterative. The number of iteration increased with network load increasing. The maximum number of iterations for this

case (for two flows) was 11 and observed when the flows rate was 100 1/s.

Let's consider second variant when tree flows circulate in the network. Maximum total rate of tree flows as well as in the previous example equals 200 1/s. However the maximum number of iterations is 7.

Let's consider third variant when four flows circulate in the network. Maximum total rate of four flows as well as in the previous examples equals 200 1/s. However the maximum number of iterations is 3.

Table II shows the results of a comparative analysis of the convergence of multicast routing for different number of flows which form the same network load. In Table II  $\lambda$  is total flow rate;  $z_2$  is number of coordinative iterations when two flows circulate in network;  $z_3$  is number of coordinative iterations when tree flows circulate in network;  $z_4$  is number of coordinative iterations when four flows circulate in network.

TABLE II  
RESULTS OF A COMPARATIVE ANALYSIS OF THE CONVERGENCE OF  
MULTICAST HIERARCHICAL ROUTING METHOD ABLE

№	$\lambda$	$z_2$	$z_3$	$z_4$
1	20	0	0	0
2	25	0	0	0
3	30	0	0	0
...	...	...	...	...
30	150	0	0	0
31	155	1	1	1
32	160	1	1	1
33	165	1	1	1
34	170	2	1	1
35	175	3	3	1
36	180	4	3	2
37	185	5	3	2
38	190	5	4	2
39	195	8	5	3
40	200	11	7	3

According to the Table II data the graph was plotted. Graph (Fig. 3) shows the dependence number of iterations on total flow rate for tree cases: when two flows, three flows and four flows circulate in network.

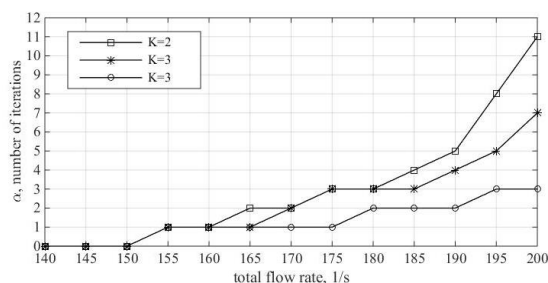


Fig. 3. The dependence number of iterations on total flow rate

As shown in Fig. 3 the number of coordinative iterations (14)-(15) essentially depends on the number of flows which form the same network load. With an increasing number of flows number of iterations reduces 2-3 times.

## V. CONCLUSION

In this paper the method of hierarchical multicast routing was analyzed. It is shown that the effectiveness of the proposed method of multicast routing greatly determines by the of coordination procedures (14)-(15) convergence rate. The less number of iterations is needed to obtain the optimum solution, the lower volume of circulating service (coordinating) traffic is in the network.

Typically, when the network load to 70-75% coordination is not required, because distributed calculation of multicast routes each border routers did not lead to links overload. But when the network load was more then 70-75% the coordination process carried iterative character and converged in a finite number of steps (1 to 11). During the study it was found that the number of coordinative iterations (14)-(15) essentially depends on the number of flows which form the same network load (Fig. 3). The number of iterations reduces 2-3 times with an increasing number of flows.

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