Performance Analysis of Tensor Approach to Multipath QoS-based Routing

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Abstract – **Tensor approach to multipath routing allows formulating analytical boundary conditions for ensuring required quality of service under multiple metrics (rate, average delay and packet loss probability) at same time. Numerical research results show implementation of the conditions allows decreasing end-to-end delay, increasing delivery rate, minimizing packet jitter caused by the multiple paths, increasing the probability of ensuring QoS-requirements.**

Keywords – **Tensor model, Multipath routing, Quality of service.**

I. Introduction

Essential attribute of modern telecommunication systems (TCS) is multiple services where every service can be provided at different levels of Quality of Service (QoS). Every grade of service has own requirements (of average delay, jitter, packet loss and packet transmission rate) which must be satisfied simultaneously. Satisfaction of the whole set of requirements is a complex theoretical and practical problems, because the end-to-end quality of service is the result of the interaction of multiple network devices, protocols and mechanisms. From this viewpoint routing is most effective tool which can coordinate network devices in order to ensure QoS requirements. As a result we have actual problem of QoS-routing which consists in finding an optimal path or paths from a source to a destination subject to all QoS-constraints (requirements) on the path. In order to improve utilization of telecommunication network's elements (routers and links) we'll focus on multipath routing.

There are two main approaches to multipath routing problem. As far as graph models formulate routing problem as searching of the shortest path on graph without handling flows we'll focus on flow models. It allows (1) load balancing between multiple paths in order to improve efficiency of TCS, (2) taking into account nature of real telecommunications trafqfic, (3) controlling QoS-parameters along all delivery paths in order to satisfy user's requirements. From viewpoint of ensuring quality of service and load balancing within flow models special attention deserves the tensor approach. It is effective means of holistic and multidimensional description of TCS. Tensor models are based on the simultaneous use of structural and functional information about the TCS that leads to improve the adequacy of the mathematical description. But on the other hand, it complicates the formalization of control network traffic and making control decisions. It is important to understand that the complexity of mathematical description must be repaid by the higher (compared to the known analogues) efficient routing decisions and higher quality of service. In this context the aim of the article is, firstly, to develop the tensor model of multipath QoS-routing, and secondly, to demonstrate the numerical benefits of the proposed approach.

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II. A Flow-based Routing Model with Packet Loss

Within the routing model structure of TCS will be described by a one-dimensional network $S=(U,V)$, where $U=\{u_i, i=\overline{1,m}\}$ is set of nodes (routers) in the network, $V=\{v_z=(i,j); z=\overline{1,n}; i,j=\overline{1,m}; i\neq j\}$ is set of edges. Here the edge $v_z=(i,j)\in V$ models z-th link connected router i to router j. Assume capacity $\varphi_{(i,j)}$ of every link (i,j) measured as number of packets per second is known.

Each router has multiple interfaces, through which it transmits packets to adjacent nodes (the neighbors). Assume the number of interface corresponds to the number of neighboring node which has been connected through the interface. The result of the routing problem solving is the calculation of the set of routing variables $x^k_{(i,j)}$, each of which characterizes the fraction of the intensity of the k-th traffic directed from the node i to the node j via an appropriate, i.e. j-th interface.

In process of the TCS functioning queue overflow raises packet losses on the interfaces of nodes. Let us use $p_{(i,j)}$ to note the probability of packet loss on the (i,j) interface. Then expression $x^k_{(i,j)}(1-p_{(i,j)})$ defines fraction of the intensity of the k-th traffic transmitted through link (i,j). But $x^k_{(i,j)}p_{(i,j)}$ is fraction of the k-th traffic dropped on j-th interface of node i.

In order to take into account the lost packets on nodes of TCS law of flow conservation must be rewritten in new form:

$$\begin{cases} \sum_{j:(i,j)\in V} x_{(i,j)}^k = 1, \ k \in K, \ i = s_k; \\ \sum_{j:(i,j)\in V} x_{(i,j)}^k - \sum_{j:(j,i)\in V} x_{(j,i)}^k (1 - p_{(j,i)}) = 0, \ k \in K, \ i \neq s_k, d_k; \\ \sum_{j:(j,i)\in V} x_{(j,i)}^k (1 - p_{(i,j)}) = \varepsilon^k, \ k \in K, \ i = d_k, \end{cases} \quad (1)$$

where K is set of traffics in the network; s_k is node-sender and d_k is node-receiver for the k-th traffic; ε^k is fraction of the k-th traffic serviced by the network, i.e. this fraction of packets from the sending node to the receiving node delivered successfully.

If take into account all possible packet losses the intensity of the traffic that is transmitted through the link $(i, j) \in V$ can be calculated as

$$\lambda_{(i,j)} = \sum_{k\in K} \lambda_k^{\langle req \rangle} x_{(i,j)}^k (1 - p_{(i,j)}), \quad (2)$$

where $\lambda_k^{\langle req \rangle}$ is average intensity of the k-th traffic that arrives into network. This value is required packet transmission rate, one of QoS-metrics.

In order to implement multipath routing and eliminate capacity overload the control (routing) variables $x_{(i,j)}^k$ must satisfy the conditions

$$0 \le x_{(i,j)}^k \le 1, \quad (3)$$

$$\lambda_{(i,j)} \le \varphi_{(i,j)}, \quad (i, j) \in E. \quad (4)$$

Because of the random nature of the network traffic performance of constraints (4) is only a necessary, but not sufficient condition for the absence of packet loss on the nodes of TCS caused by possible buffer overflow.

III. Tensor model of multipath QoS-routing based on multiple metrics

In order to obtain a tensor model of the TCS let us introduce anisotropic space-structure constructed by the set of closed and open paths (circuits and node pairs). The dimension of this space is determined by the total number of edges in the network, and is equal to n [1].

Assume S is a connected network, i.e. contains one connected component, then the cyclomatic number $\mu(S)$ and rank $\phi(S)$ of the network define the number of basic circuits and node pairs, determining the validity of the following expressions:

$$\phi(S) = m - 1, \ \mu(S) = n - m + 1, \ n = \phi(S) + \mu(S). \quad (5)$$

From all possible circuits let us choose $\mu(S)$ of linearly independent circuits $\{\pi_i, i = \overline{1,\mu}\}$; set of all node pairs we will note as $\{\eta_j, j = \overline{1,\phi}\}$. These sets form basis for n-dimension space, corresponding to the structure of the network.

In introduced n-dimension space, the TCS can be described by mixed divalent tensor

$$\mathrm{Q} = \mathrm{T} \otimes \Lambda, \quad (6)$$

where $\otimes$ is sign of direct tensor multiplication, and components of tensor Q are univalent covariant tensor of average packet delay T and univalent contravariant tensor of traffic intensity Λ.

To consider the tensor (6) we will take into account the following two coordinate systems (CS): coordinate system of the edges (type *v*) and the coordinate system of the circuits and node pairs (type $\pi\eta$). Within tensor approach one of key role belongs to the variance of used tensors. It is known that the characteristics (metrics) which are submitting to conservation law, for example, the intensity of the traffic, are contravariant values, and the additive parameters, for example, the average packet delay, are covariant values [2, 3].

The solving of routing problem requires analytical representation of the interfaces. For the formal description of the interface we will use the results of queuing theory as one possible way to do it. For instance by using queuing system $\mathrm{M/M/1/N}$ the average packet delay in i-th communication link is approximated by

$$\tau_i^v = \frac{\rho_v^i - (\rho_v^i)^{N_i+2} - (N_i + 1)(\rho_v^i)^{N_i^v+1}(1-\rho_v^i)}{(1-(\rho_v^i)^{N_i^v+1})(1-\rho_v^i)(\lambda_v^i)^2} \lambda_v^i, \quad (7)$$

where index v shows type of coordinate system (all parameters in (7) belong to i-th edge that models some link, so every parameter here belongs to coordinate system of the edges); ρ_v^i is utilization of the link; $N_i = \Theta_i + 1$ is maximal number of packets, which can be on the i-th interface, including buffer (Θ_i) and, actually, link; λ_v^i is packet intensity of the total traffic transmitted through the link, 1/s.

Note that in the Eq. (7) symbols ρ_v^i and τ_i^v belong to the same k-th traffic flow and Eq. (7) defines delay for packets from the flow. Here and hereinafter in order to simplify mathematics we'll omit index of traffic (k).

In accordance with the Kron's postulate of the second generalizations [1], a system of equations (7) can be replaced by the following vector equation

$$\Lambda_v = G_v T_v, \quad (8)$$

where Λ_v and T_v are projections of tensor Λ and tensor T in CS of edges respectively, Λ_v and T_v are n-dimensional vectors of traffic intensity and average packet delay in edges of the network respectively; $G_v = \left\| g_v^{ij} \right\|$ is $n \times n$ diagonal matrix, diagonal elements of which are calculated according to (7), i.e.

$$g_v^{ij} = \frac{(1-(\rho_i^v)^{N_i^v+1})(1-\rho_i^v)(\lambda_i^v)^2}{\rho_i^v - (\rho_i^v)^{N_i^v+2} - (N_i^v + 1)(\rho_i^v)^{N_i^v+1}(1-\rho_i^v)}. \quad (9)$$

Covariant character of delay tensor T and contravariant character of tensor Λ cause following linear laws of the coordinate transformation:

$$\Lambda_v = C\,\Lambda_{\pi\eta}\,, \qquad T_v = A\;T_{\pi\eta}\,, \tag{10}$$

where C and A are $n\times n$ matrices of contravariant and covariant coordinate transformation respectively when transition from CS of circuits and node pairs to CS of edges. The matrix A is related to matrix C by orthogonality condition $CA^t = I$; I is unity $n\times n$ matrix, $[\cdot]^t$ is transposition sign.

In (10) $\Lambda_{\pi\eta}$ is n-dimensional vector that is projection of tensor Λ in coordinate system of circuits and node pairs (type $\pi\eta$). The vector $\Lambda_{\pi\eta}$ has components

$$\Lambda_{\pi\eta} = \begin{vmatrix} \Lambda_\pi \\ -- \\ \Lambda_\eta \end{vmatrix}; \; \Lambda_\pi = \begin{vmatrix} \lambda_\pi^1 \\ \vdots \\ \lambda_\pi^j \\ \vdots \\ \lambda_\pi^\mu \end{vmatrix}; \; \Lambda_\eta = \begin{vmatrix} \lambda_\eta^1 \\ \vdots \\ \lambda_\eta^p \\ \vdots \\ \lambda_\eta^\phi \end{vmatrix}, \tag{11}$$

where Λ_π is μ-dimensional subvector related to traffic intensities into circuits of the network; Λ_η is ϕ-dimensional subvector related to traffic intensities between node pairs of the network; λ_π^j is the traffic intensity in circuit π_j of the network; λ_η^p is the traffic intensity which is entering into the network and outgoing the network through the node pair η_p.

Projection of the tensor of average delays T in a coordinate system of circuits and node pairs (type $\pi\eta$) is represented by the n-dimensional vector with the following structure:

$$T_{\pi\eta} = \begin{vmatrix} T_\pi \\ -- \\ T_\eta \end{vmatrix}; \quad T_\pi = \begin{vmatrix} \tau_1^\pi \\ \vdots \\ \tau_j^\pi \\ \vdots \\ \tau_\mu^\pi \end{vmatrix}; \quad T_\eta = \begin{vmatrix} \tau_1^\eta \\ \vdots \\ \tau_p^\eta \\ \vdots \\ \tau_\phi^\eta \end{vmatrix}, \tag{12}$$

where τ_j^π, τ_p^η are average packet delay in circuit π_j and between node pair η_p respectively. Sizes of T_π and T_η are μ and ϕ respectively.

Expression (8) has the same form in coordinate system of circuits and node pair:

$$\Lambda_{\pi\eta} = G_{\pi\eta}\,T_{\pi\eta}\,, \tag{13}$$

where $G_{\pi\eta}$ is projection of tensor G in CS of circuits and node pair, tensor G is divalent contravariant metric tensor,

$$G_{\pi\eta} = A^t G_v\,A\,. \tag{14}$$

Let us consider within the tensor model (5)-(14) the problem of conditions for QoS ensuring in TCS. Assume we have numerical requirements of average delay $\tau_{\langle req\rangle}$, packet loss probability $p_{\langle req\rangle}$ and packet transmission rate (intensity) $\lambda^{\langle req\rangle}$. Our goal is obtaining analytical conditions for ensuring all of the requirements. Let us agree that node pair source-destination will have the first number (11).

Under routing model represented by expressions (1)-(4) packet loss rate on node u_i is calculated as

$$\lambda_\eta^i = \sum_{j=1}^{R_i} \lambda_k^{\langle req\rangle} x_{(i,j)}^k p_{(i,j)}\,,$$

where R_i is number of output interfaces on node i.

Then the required conditions for QoS ensuring are

$$\sum_{j=2}^{\phi} \lambda_\eta^j \le \lambda^{\langle req\rangle} p_{\langle req\rangle}\,. \tag{15}$$

$$\lambda^{\langle req\rangle}(1-p_{\langle req\rangle}) \le G_{\pi\eta}^{\langle 4,2\rangle}\left[G_{\pi\eta}^{\langle 4,4\rangle}\right]^{-1}\Lambda_{\eta-1} + \left(G_{\pi\eta}^{\langle 4,1\rangle} - G_{\pi\eta}^{\langle 4,2\rangle}\left[G_{\pi\eta}^{\langle 4,4\rangle}\right]^{-1}G_{\pi\eta}^{\langle 4,3\rangle}\right)\tau_{\langle req\rangle}\,, \tag{16}$$

where

$$G_{\pi\eta} = \begin{Vmatrix} G_{\pi\eta}^{\langle 1\rangle} & | & G_{\pi\eta}^{\langle 2\rangle} \\ --- & + & --- \\ G_{\pi\eta}^{\langle 3\rangle} & | & G_{\pi\eta}^{\langle 4\rangle} \end{Vmatrix}; \quad G_{\pi\eta}^{\langle 4\rangle} = \begin{Vmatrix} G_{\pi\eta}^{\langle 4,1\rangle} & | & G_{\pi\eta}^{\langle 4,2\rangle} \\ --- & + & --- \\ G_{\pi\eta}^{\langle 4,3\rangle} & | & G_{\pi\eta}^{\langle 4,4\rangle} \end{Vmatrix}$$

and $G_{\pi\eta}^{\langle 1\rangle}$ are square $\phi\times\phi$ and $\mu\times\mu$ submatrices, respectively, $G_{\pi\eta}^{\langle 2\rangle}$ is $\mu\times\phi$ submatrix, $G_{\pi\eta}^{\langle 3\rangle}$ is $\phi\times\mu$ submatrix; $G_{\pi\eta}^{\langle 4,1\rangle}$ is the first element of the matrix $G_{\pi\eta}^{\langle 4\rangle}$.

Inequalities (15) and (16) are the desired conditions to ensure quality of service on set of heterogeneous metrics (transmission rate ($\lambda^{\langle req\rangle}$), time ($\tau_{\langle req\rangle}$) and reliability ($p_{\langle req\rangle}$)) simultaneously. Note, Eqs. (15) and (16) are QoS ensuring conditions for k-th traffic flow and in general such conditions must be written for every flow which requires guaranteed quality.

In addition, the derivation of the inequalities (15), (16) is based on requirement $T_\pi = 0$. It means delay in every circuit must be zero. And it guarantees that delay in pair source-destination will be minimal and delays along different paths will be same. As a result jitter caused by multipath routing is minimized.

IV. Performance Analysis of Tensor Model

In order to demonstrate the advantages of the proposed tensor model we compared it with another known mathematical models, in which multi-path routing is formulated as an optimization problem with different optimality criteria. A brief description of the investigated models (M1-M3) is following:

- M1 has criterion $\min \max_i \left\{ \frac{\lambda^i_{(v)}}{\varphi_i} \right\}$ subject to (1)-(4),

- within M2 criterion is $\min \max_i \{q_i\}$ subject to (1)-(4), where q is average length of queue on i -th interface

- M3 is optimization problem $\min \max_i \left\{ \frac{\lambda^i_{(v)}}{\varphi_i} \right\}$ subject to (1)-(4), (15), (16).

Example of model M3 implementation is shown in Fig. 1. For given QoS-requirements $\lambda^{\langle req \rangle} = 160$ 1/s, $\tau_{\langle req \rangle} = 92$ ms and $p_{\langle req \rangle} = 3 \cdot 10^{-2}$ we defined set of used paths and traffic allocation between them that provides required quality.

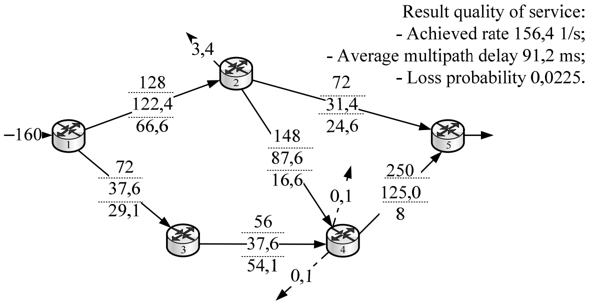

Fig. 1. Example of network and calculated traffic allocation

As numerical research results show M3 provides gain in end-to-end delay compared to the models M1 and M2 to a value between 15 and 40% at the same intensity of arriving traffic and loss value (Fig. 2). The M3 model provides higher rate (an average of 40-50% under the low and medium load, and at least 10-20% under high load, Fig. 2).

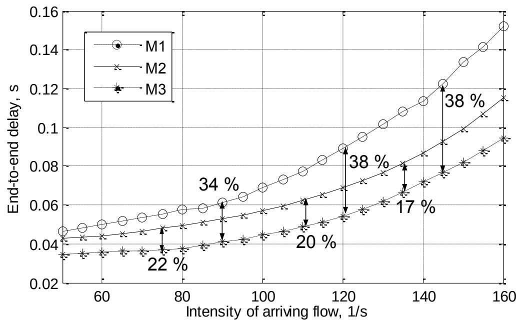

Fig. 2. Delay dependence on intensity of arriving traffic

As expected, the value of jitter caused by multipath routing in accordance with the models M1 and M2 increases with the intensity of the incoming flow, while for the model M3 jitter is practically absent (Fig. 3).

As far as the routing problem is one of the most effective means to ensure the required quality of service, the efficiency of routing model can be evaluated by probability of ensuring QoS-requirements. It's the probability of satisfying all of QoS-requirements. As Fig. 4 shows applying the model M3 can increase the probability, for example, for the structure Fig. 2 up to 17% compared to the model M1, and up to 50% compared to the model M2 under the same QoS-requirements.

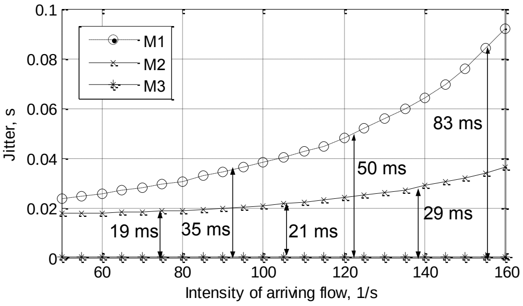

Fig. 3. Jitter dependence on intensity of arriving traffic

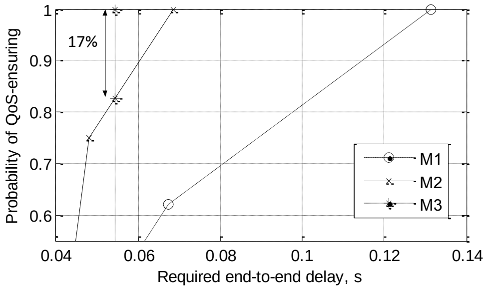

Fig. 4. Probability of QoS-ensuring

III. Conclusion

The tensor model of multipath QoS-routing allows obtaining the analytical conditions for ensuring transmission rate, average delay and packet loss probability simultaneously. The results of numerical analysis of the proposed and previously known solutions demonstrated the benefits of tensor approach to routing problem. Implementation of proposed tensor model allows decreasing end-to-end delay (up to 15-40%) or increasing the transmission rate (up to 10-50%), minimizing packet jitter caused by the multiple paths, increasing the probability of ensuring QoS-requirements.

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