About One Class of Vector Random Processes With Infinite Rank of Nonstationarity

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***Abstract* — One class of nonstationary vector random processes is investigated in the article. On the basis of triangle models of quasiunitary operators the structure of corresponding correlation matrix was investigated, representations for special matrix were got, by which the correlation matrix is retrieved after solving the Darboux-Goursat problem.**

***Index Terms*— correlation matrix of nonunitarity; triangle model of Cayley transform; evolutionary interpretable vector curve in Hilbert space**

I. Introduction

LET us consider a vector non-stationary random process (VNRP) of the type $\xi(t) = (\xi_1(t,\omega),\ \xi_2(t,\omega))$ ($\omega \in \Omega$ (probabilistic space) and $t \in [0,\infty)$) with $M\xi_j(t,\omega) = 0$ and continuous correlation matrix (CM) $K_{\alpha\beta}(t,s) = M\xi_\alpha(t,\omega)\overline{\xi_\beta(s,\omega)}$. VNRP after embedding into the Hilbert space $H = \overline{\bigvee_{\substack{k,j\\ j=\overline{1,n}}} c_{k,j}\xi_j(t_k)}$ will result in vector curve in H of the type $x(t) = (x_1(t), x_2(t))$, at the same time the elements of CM can be calculated as corresponding scalar products in H: $K_{\alpha\beta}(t,s) = \langle x_\alpha(t), x_\beta(s)\rangle$ [1-3].

In the following with purpose of simplification we will suppose, that $H_j = \overline{\bigvee_k c_{k,j}\xi_j(t_k)} = H$ ($j = 1, 2$).

The case of uncorrelated $\xi_1(t)$ and $\xi_2(t)$ is of little interest, as in this case $H = H_1 \oplus H_2$, and $\left(K_{\alpha\beta}(t,s)\right) = \begin{pmatrix} K_{11}(t,s) & 0 \\ 0 & K_{22}(t,s)\end{pmatrix}$, and practically reduce to scalar variant [3].

If additional conditions for correlation matrix [4], are fulfilled, the curve $x(t)$ is the following $x(t) = (e^{tT_1}x_{01}, e^{tT_2}x_{02})$, where T_k ($k = 1, 2$) are linear bounded operator in H.

The case when T_j have the form of iA_j, and $\dim\overline{(A_\alpha - A_\beta^*)H} < \infty$ is considered in [4].

If $T_1 = T_2 = iA$, where A is the selfadjoint operator, then we will have stationary vector curve [5, 6].

In this article the T_j is investigated, which slightly deviates from unitary ones in the meaning that $\dim\overline{(I - T_j^*T_i)H} < \infty$, ($i, j = \overline{1,2}$) [7, 8].

In case when $T_1 = T_2 = U$, where U is a unitary operator, the rank of nonstationarity corresponding to the vector evolutionary interpretable curve $(e^{tT}x_{01}, e^{tT}x_{02})$ is equal to infinity.

Purpose of this work is setting numerical and functional characteristics allowing to describe deviations of vector random process from the process, which is represented in corresponding Hilbert space as $(e^{tU}x_{01}, e^{tU}x_{02})$, where U is a unitary operator.

Definition. The vector curve $(e^{tU}x_{01}, e^{tU}x_{02})$ will be designated as *unitary*.

It is easy to get a spectral representation of such curve and corresponding correlation matrix using spectral decomposition of unitary operator $U = \int_0^{2\pi}\lambda dE_\lambda$ (E_λ is a resolution of identity):

$$x(t) = \left(\int_0^{2\pi}\exp(te^{i\lambda})d\eta_1(\lambda), \int_0^{2\pi}\exp(te^{i\lambda})d\eta_2(\lambda)\right),$$

where $\Delta\eta_j(\lambda) = \Delta E_\lambda x_{0j}$ are processes with uncorrelated increments, and the corresponding correlation matrix is the following

$$K_{\alpha\beta}(t,s) = \int_0^{2\pi}\exp(te^{i\lambda} + se^{-i\lambda})dF_{\alpha\beta}(\lambda) =$$
$$= \int_0^{2\pi}e^{(t+s)\cos\lambda}(\cos((t-s)\sin\lambda) +$$
$$+ i\sin((t-s)\sin\lambda))\, dF_{\alpha\beta}(\lambda) = K_{\alpha\beta}(t-s, t+s),$$

where

$$\Delta F_{\alpha\beta}(\lambda) = \langle\Delta\eta_\alpha(\lambda), \Delta\eta_\beta(\lambda)\rangle_H = M\Delta\eta_\alpha(\lambda)\overline{\Delta\eta_\beta(\lambda)}.$$

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II. STATEMENT OF PROBLEM

It is evident that infinitesimal correlation matrix with components $(\partial t+\partial s)K_{\alpha\beta}(t,s)$ does not describe the structure of deviation of vector curve from unitary curve. Other characteristic should be introduced, which in case of unitary evolutionary interpretable vector curve will be identically equal to 0. Such characteristic can be matrix-function $\hat{N}(t,s)$ with elements of the following type:

$$N_{\alpha\beta}(t,s)=\frac{\partial^2 K_{\alpha\beta}(t,s)}{\partial t\partial s}-K_{\alpha\beta}(t,s), \qquad (1)$$

which we will designate as *a correlation matrix of nonunitarity*.

It is clear that for unitary curve $N_{\alpha\beta}(t,s)\equiv 0$.

The recovery of correlation matrix $K_{\alpha\beta}(t,s)$ demands solution of equation in partial derivatives of telegraphic type:

$$\frac{\partial^2 K_{\alpha\beta}(t,s)}{\partial t\partial s}-K_{\alpha\beta}(t,s)=N_{\alpha\beta}(t,s),$$

by additional conditions: $K_{\alpha\beta}(t,0)=F_{\alpha\beta}(t)$, $K_{\alpha\beta}(0,s)=G_{\alpha\beta}(s)$, $F_{\alpha\beta}(t)=\overline{G_{\beta\alpha}(t)}$, i.e. it comes to the Darboux-Goursat problem.

One of the most important problems of correlation theory of random processes is getting spectral representations for processes and corresponding correlation matrixes. Using of (1) and triangle models of quasiunitary operators [7, 8] allows to get model representations for $N_{\alpha\beta}(t,s)$, and accordingly for CM as solution of corresponding Darboux-Goursat problem for equations in partial derivatives of hyperbolic type. This work is intended for getting such representations.

III. SOLUTION OF PROBLEM.

For evolutionary representable vector curve in H_ξ it is easy to get the following expressions for $N_{\alpha\beta}(t,s)$:

$$N_{\alpha\beta}(t,s)=\left\langle (I-T_\alpha^* T_\beta)x_\alpha(t),\, x_\beta(s)\right\rangle_H. \qquad (2)$$

Theorem 1. In order that vector evolutionary interpretable curve $x(t)=(e^{tT_1}x_{01}, e^{tT_2}x_{02})$ ($\|T_k\|<\infty$) will be a unitary one, it is necessary and enough that $N_{\alpha\beta}(t,s)\equiv 0$.

Proof. Necessity is evident, for adequacy it should be taken in consideration that from (1) follows

$$\left\langle (I-T_\alpha^* T_\beta)\sum_{j=1}^{n_1} a_j x_\alpha(t_j),\, \sum_{l=1}^{n_2} b_l x_\beta(s_l)\right\rangle_H \equiv 0 .$$

Turning to closure we will have

$$\left\langle (I-T_\alpha^* T_\beta)h_1,\, h_2\right\rangle = 0$$

for each $h_1, h_2\in H$, i.e. $I-T_\alpha^* T_\beta=0$.

From this follows that T_j are unitary operators (supposed that $\alpha=\beta$).

By $\alpha\neq\beta$ ($\alpha=1,\beta=2$) $I-T_1^* T_2=0$, but as $T_1^*=T_1^{-1}$, it follows from this that $T_1=T_2$.

Definition. Maximal rank of quadratic forms

$$\sum_{k,j=1}^{m}\left\langle \hat{N}(t,s)\vec{z}_k,\, \vec{z}_j\right\rangle \ (m=1,2,...)$$

is determined as *nonunitary index*, where $\vec{z}_k=\begin{pmatrix} z_k^{(1)} \\ z_k^{(2)}\end{pmatrix}$, $z_k^{(j)}$ are arbitrary complex numbers.

The following theorem is the criterion of non-unitary index finiteness.

Theorem 2. In order that non-unitary index of evolutionary interpretable vector random process will be a finite one, it is necessary and enough that $\dim\overline{(I-T_j^* T_i)H}<\infty$.

Proof of evidence is similar to the proof of theorem 3 of the work [4] and therefore it is left out.

In the following we will be confined by investigation of non-unitary vector curves of the type

$$x(t)=(e^{tT}x_{01}, e^{tT}x_{02}),\ \|T\|<\infty,\ T^*T\neq I. \qquad (3)$$

which we will define as a *primary nonunitary vector random processes.*

For evolutionary interpretable non-unitary vector process it is easy to get the coincidence criterion of T_1 and T_2 (3).

In fact, we will suppose that T_2^{*-1} exists. Let us consider a random scalar process $x_2^*(t)=\exp(-tT_2^{*-1})x_{02}$.

We will find a cross-correlation function $x_1(t)$ and $x_2^*(t)$: $K_{12}^*(t,s)=\left\langle e^{tT_1}x_{01}, e^{sT_2^{*-1}}x_{02}\right\rangle$. Now we will calculate $N_{12}^*(t,s)=\frac{\partial^2 K_{12}^*(t,s)}{\partial t\partial s}-K_{12}^*(t,s)$. From the definition $K_{12}^*(t,s)$ it follows that

$$N_{12}^*(t,s)=\left\langle (T_2^{-1}T_1-I)x_1(t),\, x_2^*(s)\right\rangle.$$

It is evident the following: if $T_2=T_1$, then $N_{12}^*(t,s)\equiv 0$.

On the other hand if $N_{12}^*(t,s)\equiv 0$, then by reasoning in similar way like by proof of theorem 1 we will have $T_2^{-1}T_1-I=0$, so $T_1=T_2$. In such way we will have the following criterion.

Theorem 3. In order that vector non-stationary evolutionary interpretable random process

$(e^{tT_1}x_{01}, e^{tT_2}x_{02})$ will be a primary one, it is necessary and enough that $N^*_{12}(t,s) \equiv 0$.

We will establish bond between $K^*_{12}(t,s)$ and

$$K_{12}(t,s) = \langle x_1(t), x_2(s)\rangle_H = M\xi_1(t)\overline{\xi_2(s)}.$$

We should note that $x^*_2(t) = e^{tT_2^{*-1}}x_{02}$ is a solution of Cauchy problem:

$$\begin{cases} \dfrac{dx^*_2(t)}{dt} = T_2^{*-1}x^*_2(t), \\ x^*_2(0) = x_{02}. \end{cases} \tag{4}$$

In the following with purpose of simplification we will suppose, that

$$I - T_2^*T_2 = \langle \cdot, g\rangle g, \tag{5}$$

i. e. in T_2 is a one-dimensional value of non-unitary component (it is not difficult to integrate in case of $\dim\overline{(I - T_2^*T_2)H} = \rho$ $(2 \le \rho < \infty)$).

After changing in (4) T_2^{*-1} to T_2 from (5) ($T_2^{*-1} = T_2 + \langle \cdot, g\rangle g_1, g_1 = T_2^{*-1}g$), we will have the following:

$$\frac{dx^*_2}{dt} = T_2x^*_2 + \left\langle x_{02}, e^{tT_2^{-1}}g\right\rangle g_1. \tag{6}$$

We will define $\Lambda_g(t) = e^{tT_2^{-1}}g$ – Λ_g *as a curve* for operator T_2^{-1} with one-dimensional non-unitary component; $\Lambda_g(t)$ will be recovered by spectrum with precision to unitary equivalence [11].

Solution of equation (6) is the following

$$x^*_2(t) = x_2(t) + \int_0^t \langle x_{02}, \Lambda_g(u)\rangle \Lambda_{g1}(t-u)du,$$

where $\Lambda_{g1}(t) = e^{tT_2}g_1$.

We will note that Λ_{g1} curve can also be recovered only by spectrum with precision to unitary equivalence.

Then for $K^*_{12}(t,s)$ we will get the representation:

$$K^*_{12}(t,s) = K_{12}(t,s) + \\ + \int_0^s \langle x_{02}, \Lambda_g(u)\rangle\langle e^{tT}x_{01}, \Lambda_{g1}(s-u)\rangle du. \tag{7}$$

If $g = 0$, i. e. $T_2^* = T_2^{-1}$, then $K^*_{12}(t,s) = K_{12}(t,s)$.

The representation (7) for $K^*_{12}(t,s)$ can be considered as a "canonical" one, as by transfer to unitary equivalent elements the scalar products will not be changed.

Notice. If we will consider the deviation of vector random process from the stationary one, i.e. by consideration of the process of the type $(e^{itA_1}x_{01}, e^{itA_2}x_{02})$, then for $(e^{itA}x_{01}, e^{itA}x_{02})$, where $A \ne A^*$, it is necessary and enough that

$$W^*_{12}(t,s) \equiv 0,$$

where $W^*_{12}(t,s) = -(\partial t + \partial s)\left\langle e^{itA_1}x_{01}, e^{isA_2^*}x_{02}\right\rangle$.

Now let us consider linear continuous systems, associated with unitary operator unit [3, 8–10], containing a non-unitary operator T of the type

$$\begin{cases} \dfrac{dx_\alpha(t)}{dt} = Tx_\alpha(t) + \Phi u_\alpha, \\ v_\alpha = Ku_\alpha + \Psi x_\alpha(t), \\ x_\alpha(0) = x_{\alpha 0}, \end{cases} \tag{8}$$

where $x_\alpha \in H$, $u_\alpha \in E$, $v_\alpha \in F$, and linear mappings $T \in [H,H]$, $\Phi \in [E,H]$, $K \in [E,F]$, $\Psi \in [H,F]$ satisfy relations of unitary operator unit [9], which represents condition of operator matrix unitary $D = \begin{pmatrix} T & \Phi \\ \Psi & K \end{pmatrix}$ (taking into consideration indefinite metric in external spaces E and F), $D^+D = I$: $\begin{pmatrix} T^* & \Psi^+ \\ \Phi^+ & K^+ \end{pmatrix}\begin{pmatrix} T & \Phi \\ \Psi & K \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$:

$$T^*T + \Psi^+\Psi = I,$$
$$T^*\Phi + \Psi^+K = 0,$$
$$\Phi^+T + K^+\Psi = 0,$$
$$\Phi^+\Phi + K^+K = I.$$

(the remaining 4 ratios resulting from the condition $DD^+ = I$ are not used in the following and not stated).

We should note that in case when T^{-1} exists, the spaces F and E can be chosen as coincident, and mapping Φ and Ψ as independent [10].

Theorem 4. The matrix-function $N_{\alpha\beta}(t,s)$ (1) is expressed by correlation matrixes of input and output signals in the following way:

$$N_{\alpha\beta}(t,s) = [u_\alpha, u_\beta]_E - [v_\alpha, v_\beta]_F,$$

where the indefinite scalar product in spaces E and F correspondingly is stated in square brackets.

The proof follows from the next chain of ratios based on equations of associated opened system (8):

$$N_{\alpha\beta}(t,s) = \frac{\partial^2 K_{\alpha\beta}(t,s)}{\partial t\partial s} - K_{\alpha\beta}(t,s) =$$
$$= \left\langle \frac{dx_\alpha(t)}{dt}, \frac{dx_\beta(s)}{ds}\right\rangle - \langle x_\alpha(t), x_\beta(s)\rangle =$$
$$= \langle Tx_\alpha(t) + \Phi u_\alpha, Tx_\beta(s) + \Phi u_\beta\rangle - \langle x_\alpha(t), x_\beta(s)\rangle =$$
$$= \langle (T^*T - I)x_\alpha(t), x_\beta(s)\rangle + [\Phi^+Tx_\alpha(t), u_\beta] +$$
$$+ [u_\alpha, \Phi^+Tx_\beta(s)] + [\Phi^+\Phi u_\alpha, u_\beta] =$$

$$= \left\langle -\Psi^{+}\Psi x_\alpha(t), x_\beta(s) \right\rangle - [\Psi x_\alpha(t), Ku_\beta] -$$
$$- [Ku_\alpha, \Psi x_\beta(s)] + [u_\alpha, u_\beta] - [Ku_\alpha, Ku_\beta] =$$
$$= [-\Psi x_\alpha(t), \Psi x_\beta(s)] - [v_\alpha - Ku_\alpha, Ku_\beta]_F -$$
$$- [Ku_\alpha, v_\alpha - Ku_\beta]_F + [u_\alpha, u_\beta]_E - [Ku_\alpha, Ku_\beta]_F =$$
$$= -[v_\alpha - Ku_\alpha, v_\beta - Ku_\beta]_F - [v_\alpha, Ku_\beta]_F - [Ku_\alpha, v_\beta]_F +$$
$$+ [Ku_\alpha, Ku_\beta]_F + [u_\alpha, u_\beta]_E = [u_\alpha, u_\beta]_E - [v_\alpha, v_\beta]_F.$$

As a result we have the following ratio:

$$N_{\alpha\beta}(t,s) = [u_\alpha, u_\beta]_E - [v_\alpha, v_\beta]_F.$$

In the following, the most interesting for applications is the case when $\dim E < \infty$, $\dim F < \infty$. In this case

$$[v_\alpha(t), v_\beta(s)] = \left\langle \begin{pmatrix} I_r & 0 \\ 0 & -I_s \end{pmatrix} v_\alpha, v_\beta \right\rangle =$$
$$= K^{v+}_{\alpha\beta}(t,s) - K^{v-}_{\alpha\beta}(t,s), \quad r + s = \dim F;$$
$$[u_\alpha(t), u_\beta(s)] = \langle J_E u_\alpha, u_\beta \rangle = \left\langle \begin{pmatrix} I_p & 0 \\ 0 & -I_q \end{pmatrix} u_\alpha, u_\beta \right\rangle =$$
$$= K^{u+}_{\alpha\beta}(t,s) - K^{u-}_{\alpha\beta}(t,s), \quad p + q = \dim E.$$

Therefore $N_{\alpha\beta}(t,s)$ is equal to difference of corresponding correlation matrixes of input and output signals.

Now we will consider T – a quasiunitary operator with one-dimensional deficient subspace $\dim \overline{(I - T^*T)H} = 1$, i. e. $(I - T^*T) = \langle \cdot, g \rangle g$, where g is a channel element T [8, 9].

Then for primary non-unitary vector evolutionary interpretable random process $N_{\alpha\beta}(t,s)$ can be presented as

$$N_{\alpha\beta}(t,s) = \psi_\alpha(t)\overline{\psi_\beta(s)}, \tag{9}$$

where $\psi_\alpha(t) = \left\langle e^{tT} x_{0\alpha}, g \right\rangle = \left\langle x_{0\alpha}, e^{tT^*} g \right\rangle$.

For investigation the structure $\psi_\alpha(t)$ we can use triangle models of quasiunitary operators [7, 8, 12].

Going to (9) to unitary equivalent elements we will have the following

$$\psi_\alpha(t) = \left\langle \hat{x}_{0\alpha}, \exp(t\hat{T}^*)\hat{g} \right\rangle_{\hat{H}}, \tag{10}$$

where $\hat{x}_{0\alpha} = Ux_{0\alpha}$, $\hat{T}^* = UT^*U^{-1}$, $\hat{g} = Ug$, and $U \in [H, \hat{H}]$ is a unitary operator.

Taking into consideration that by some additional conditions the operator T is unitary equivalent to its triangle model [12], in the representation (10) a triangle model can be taken as $\hat{T}^*$, it allows to get model representations for $\psi_\alpha(t)$, and correspondingly for $N_{\alpha\beta}(t,s)$ for different cases of spectrum.

So in case when the operator T has only a discrete spectrum $\{\mu_k\}$, located inside the unit circle of plane of complex numbers (constriction), the model space $\hat{H}$ is coincides with $\ell^2(\beta_k^2)$, and $\psi_\alpha(t)$ has representation $\psi_\alpha(t) = \sum_{k=1}^{\infty} x_{0\alpha}(k)\Lambda_k(t)$, where functions $\Lambda_k(t)$ are reestablished only by spectrum $\{\mu_k, |\mu_k| < 1\}$, ($\mu_k = \dfrac{1 - \overline{\lambda_k}}{1 - \lambda_k}$, where $\lambda_k = \alpha_k + i\dfrac{\beta_k^2}{2}$ ($\sum \beta_k^2 < \infty$) is a discrete spectrum of dissipative operator $\hat{A}$ in Cayley transform $T = (A - iI)(\hat{A} + iI)^{-1}$) of discrete triangle model of the operator $\hat{T}$:

$$\Lambda_k(t) = (\exp(t\hat{T}^*)g)_k = \overline{\frac{-1}{2\pi i} \oint_\gamma e^{t\lambda} \left((\hat{T}^* - \lambda I)^{-1} g\right)_k d\lambda}, \tag{11}$$

where γ includes all the spectrum of the operator $\hat{T}$.

Using the discrete part of the triangle model of the operator $\hat{T}$ and supposing with purpose of simplification that $\mu_k \neq \mu_j$, $k \neq j$, calculating the resolvent value on channel element, it is easy to get the following representations for $\Lambda_k(t)$:

$$\Lambda_k(t) = \overline{\frac{-1}{2\pi i} \sqrt{1 - |\lambda_k|^2} \oint_\gamma \frac{e^{t\lambda}}{\lambda - \overline{\lambda}_k} \prod_{j=1}^{k-1} \frac{1 - \lambda\overline{\lambda}_j}{\lambda - \overline{\lambda}_j} \cdot \frac{|\lambda_j|}{\overline{\lambda}_j} d\lambda}.$$

For continuous spectrum the model space $\hat{H}$ coincides with $L^2_{[0,1]}(\sigma(x))$ and for $\psi_\alpha(t)$ we can get the following representation $\psi_\alpha(t) = \int_0^1 x_{0\alpha}(u)\Lambda(u,t)du$, where $\Lambda(u,t)$ is the following

$$\Lambda(u,t) = \overline{\frac{-1}{2\pi i} \oint_\gamma e^{t\lambda} (\hat{T}^* - I\lambda)^{-1} f(u) d\lambda}, \tag{12}$$

where $\hat{T} = (\hat{A} - iI)(\hat{A} + iI)^{-1}$ is a Cayley transform of the operator $\hat{A}$ ($\hat{A}f = \alpha(x)f(x) + i\int_x^\ell f(t)dt$).

The representations (11), (12) we will designate as canonical ones. By different representations relatively $\alpha(x)$ ($\alpha(x) = 0$, $\alpha(x) = \alpha_1$, $\alpha(x) = x$, $\alpha(x) = \begin{cases} \alpha_1, & 0 \le x < x_1, \\ \alpha_2, & x_1 \le x < \ell \end{cases}$ etc.) we can get explicit expressions for $\Lambda(u,t)$ by the way of special functions.

IV. Conclusion

By means of triangle models of quasiunitary operators the representations for non-unitary matrix were got, which is determined by the spectrum of quasiunitary operator only, located inside the circuit of unit radius in plane of complex numbers and infinite-fold spectrum located on unit circumference.

Scientific novelty. A new class of non-stationary random processes was introduced, which is described by means of new characteristics of nonstationarity – correlation matrix of nonunitarity $N_{\alpha\beta}(t,s)$. The "conservation law" for matrix $N_{\alpha\beta}(t,s)$ was obtained too, which connects this matrix with correlation matrixes of input and output random signals of the corresponding associated opened system.

Perspectives of investigation. The method proposed in the article can be used for obtaining spectral decompositions of non-stationary vector curves, and for modeling of correlation functions of non-stationary vector random signals. Model representations for $N_{\alpha\beta}(t,s)$ in case of $\dim \overline{(I-T^*T)H} = r$ $(2 \le r < \infty)$ can be got using universal models of constrictions.

Practical significance of the work consists in the fact that the approach proposed in the article allows to construct models of nonstationary vector random signals on the spectrum only, using triangle and universal models of quasiunitary operators.

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