Functional Management Model in Peer-to-Peer Networks

Vladimir Popovsky, Kateryna Popovska
Telecommunication systems department
Kharkiv National University of Radio Electronics
Kharkiv, Ukraine
tkc2006@ukr.net

***Abstract*—For rapidly progressive type of P2P-TV video transmission technology a rather general discrete-continuous mathematical model of video transmission in the form of differential equation of state was offered. On the basis of the generalized Lagrange equation the possibility of constructing a procedure to optimize the management of fragmentation is shown.**

Keywords—peer-to-peer; TV video; Lagrange equation; fragments; differential model.

I. Overview

In recent years there has been a steady increase in user traffic, which is about 80 petabytes/month, more than half of which belongs to video traffic. The use of P2P technology has particularly rapid growth, especially P2P-TV, designed to view video content over the Internet-TV. There are many varieties of the popular P2P networks: Bit Torrent Live, Torrent Stream, PPLive, UUSee, SopCast, etc. [1].

A feature of P2P networks is the procedure for engaging other client machines of users, from which downloading certain video fragments is done. In the live-streamed video TV content is distributed to all users in real time. During the process of providing services, a number of decisions are adopted and a large number of controls are implemented: peer selection, connection and disconnection of a peer, information exchange between peers, evaluation of delays, buffer organization, and fragments planning. Appropriate procedures are performed by a variety of controlling machines: a server, global and local trackers, etc. A scheduler provides the necessary coordination of all the punctures and controls in accordance with their algorithms and chosen models.

The article presents an attempt to demonstrate the ability to select a unified mathematical model of $x(t)$-process of peer-to-peer network functioning and associated universal procedure of optimal control in the P2P-TV technology. The process of downloading a certain file $x(t)$ is accompanied by its splitting up into separate fragments derived generally from different peers. Scheduling of fragmentation process is carried out in such a way that services with the required quality are provided with the minimum time $t \in T = [t_1, t_F]$. The process $x(t)$ itself on the interval T is represented as a controlled sequence of individual files with independent values at the boundaries of the joint. This task belongs to the trunk class when transport management is carried out on selected areas (steps), and at each step the best mode of speed, minimization of delays and the level of costs is selected [2,3].

II. Description of the Functional Mathematical Model

As a result of downloading the file, we have a general discrete-continuous (DC) process x_n^{∂}, which is a class of degenerate processes possessing the sequence of discontinuity points. The solution of equation of state $dx(t)/dt$ is characterized by an unlimited number of values of derivatives [4,5]. A typical method of solving such DC problems is to replace the sequence $x_n^{\partial}(t)$ with the certain approximating spreading sequence $x_n^{\partial}(k,t)$ [6] allowing to implicitly form the operator of discrete component x_{∂} on the set of continuous action values.

The process of file formation can be written as a sequence of continuous sections filling the space between (K) and $(K)+1$- adjacent boundaries of fragments. The model of such DC process during downloading the file $[t_1, t_F]$ can be represented in the form of controlled difference equation of state [7]:

$$x(k+1) = f(k, x(k), u(k)), \qquad (1)$$

where $K \in K = 1,2,...K_F$, $u(k) \in U(k, x(k))$.

Such discrete-continuous models (DCMs) in related work [6] are represented as a two-level structure characterized by the sequence of discrete values $x(k) = x_{\partial}$, $K = 1,2,...$, interpreted as a process of the upper-level and lower-level sequence of the continuous process $x_n(t)$ filling the intervals between recurrent discrete $x(k)$ and $x(k+1)$. The transition process from the lower to the upper level takes place through the border

$$\gamma = (t_1, x_n(t_1), t_F, x_n(t_F)).$$

Managing the process $x_n^{\partial}(k)$ should be considered as two separate functions:

$u_{\partial}(k)$ - control of discrete fragments,
$u_n(t)$ - managing continuous data flow of the TV-content.

This hybrid process $x(k+1)$ is degenerate due to interruptions at the junction of fragments. To concretize this model let us represent the management of fragments sequence in the form of values discreteness u_{∂}, and the transfer of image data - as a piecewise smooth process

$$dx_n(t)/dt = f_n(z, t, x_n, u_n),$$
$$t \in T(z) = [t_1(z), t_F(z)]. \qquad (2)$$

In addition, management $u_n(k,t)$ is a piecewise-continuous function

$$u_n \subset U_n(z,t,x_n)\,,$$

where $t \in T(z) = [t_1(z)...t_F(z)]$, $z = (k, x, u_\partial)$, $k \in K'_n \subset K$ is a set of variables that play a role of parameters in the implementation of a continuous control inside fragments.

When passing x_n through the border of neighboring fragments the transition operator γ_n is used. Due to this operator the value of derivative (2) takes the form $f_n(z,t,x_n,u_n) = \theta(z,\gamma_n)$.

The model (1) and its corresponding concretizations (2), (3) allow to solve the problem of optimizing the control process [8]. Management can be constructed using various iterative procedures.

The process of fragments sequence delivery to consumer is regarded as a component of the general process (1). It lies at the basis of P2P-TV network functioning and it is represented as a discrete control model, the state of which is determined at each of k-steps

$$x_\partial(k+1) = A(k,u_\partial)x_\partial(k) + B(k,u_\partial)\,, \qquad (3)$$

where $x_\partial \in R^{m(k)}$; $u_\partial \in U(k) \subset R^{p(k)}$; $k \in K = \{k_1,...k_F\}$.

Discrete component of the process $x_\partial(k)$ is the final sequence of fragments (k_F) for the downloaded file derived from different peers according to the chosen plan.

Management $u_n(t)$ provides fragmentation process, the choice of fragments length, their corresponding arrangement and correct "stitching", which allows the user to watch a continuous smooth image.

III. Optimization of Control Process in Peer Network

The object of control $u(t)$ carried out within each of m-fragment, is the control of frames, control of rate and minimization of delay.

The task of managing the process of downloading is generally a multiobjective, multistep procedure defined on a finite set of steps k_F. The solution of problem with the above restrictions and selected models (1), (2), (3) reduces to minimizing the functional

$$J = \Phi(x(k_F),u(k_F))$$

or under appropriate linearization:

$$J = \phi(u(k_F)x(k_F))\,.$$

Optimal management of such discrete-continuous process is found with help of the generalized Lagrangian [5,6]

$$L = G(x(k_F)) - \sum_{K\backslash K'\backslash t_F} R(k,x(k),u(k) + \sum_{K_1}^{K}(G^n(z(k),\gamma^n(k)) +$$
$$+ \int_{T(z(k))} (\mu^n(z(k) - R^n(z(k),t,x^n(k,t)u^n(k,t)))dt)\,, \quad (4)$$

where (φ,φ^n) is a pair of arbitrary functions, ensuring the expansion of discrete and continuous processes,

$$G(x) = \Phi(x) + \varphi(k_F, x(k_F)) - \varphi(k_1, x(k_1))\,,$$
$$R(k,x,u) = \varphi(k+1, f(k,x,u)) - \varphi(k,x)\,,$$
$$G^n(z,\gamma^n) = -\varphi(k+1, f(k,x,u) + \varphi(k,x(k)) + \varphi^n(z,t,x_F^n) -$$
$$-\varphi^n(z,t_1,x_1^n) - \int_{T(z)} \mu^n(z,t)dt\,,$$
$$R^n(z,t,x^n,u^n) = \varphi^n(T,x^n)f^n(z,t,x^n,u^n) + \varphi^n(z,t,x^n)\,,$$
$$\mu^n = (z,t) = \sup\{R^n(z,t,x^n,u^n)\},$$
$$l(k) = \inf\{G^n(z,\gamma^n)\},$$
$$z = (k,x,u^\partial)\,.$$

K' is a subset of the approximate values in the set of continuous process $x_n(t), K' \subset K$.

As a result of the optimization procedure it is possible to find the minimum of functional L. It is shown [5] that $\min J(k,x,u)$ is simultaneously provided.

To solve (4) it is necessary to set a couple of (φ,φ_n) ensuring fulfillment of the condition [5]

$$\inf L = 0\,,$$

for the entire set of control functions u_∂, u_n and the corresponding set of states of the upper x_∂ and lower x_n level. These requirements provide specific conditions such as Bellman optimality procedures and building effective iterative procedure [9].

IV. Conclusion

The universal enough discrete-continuous differential model of processes of P2P-TV-content downloading is obtained taking into account the processes of fragmentation, fragments flow and data flow control. The possibility of constructing optimal control procedures on the basis of a generalized Lagrange solution is shown.

References

[1] Cisco Visual Networking Index: Forecast and Methodology, 2012 – 2017. Cisco Public, 2013.

[2] Z.-H. Lu, Y. Wang, and Y.R. Yang, "An Analysis and Comparison of CDN-P2P-hybrid Content Delivery System and Model," JCM, vol. 7(3), pp. 232-245, March 2012.

[3] S. Tarkoma Overlay Networks: Toward Information Networking. – CRC Press, 2010.

[4] V.F. Krotov Global Methods in Optimal Control Theory. Marcel Dekker. New York: 1996.

[5] V.I. Gurman, and I.V. Rasina, "Discrete-continuous Representations of Impulsive Processes in the Controllable Systems," Autom. Remote Control, vol. 73. No.8, pp. 1290-1300, 2012.

[6] V.I. Gurman, and N.M. Kang, "Degenerate Problems of Optimal Control," I, II, III, Autom. Remove Control, vol. 72. No.4, p. 727-739; No.5. pp. 929-943, 2011.

[7] V. Popovskij, A. Barkalov, and L. Titarenko, "Control and Adaptation in Telecommunication Systems, Springer-Verlag Berlin Heidelberg, 2011.

[8] J. Crowcroft, E. Lua, and M. Pias, "A Survey and Comparison of Peer-to-Peer Overlay Network Schemes", ICST, 2004.

[9] Van der Shaft A.J., Schumacher H. An Introduction to Hybrid Dynamical Systems: Springer-Verlag, London Ltd, 2000.