Dynamic Presentation of Tensor Model for Multipath QoS-Routing

Oleksandr Lemeshko, Oleksandra Yeremenko

Abstract - **Dynamic presentation of tensor model for multipath routing with QoS guarantees over the multiple parameters proposed. Expressions for different queuing systems modeling state of the network router interface used for representing the average packet delay as time-varying function. Conditions of ensuring QoS for the set of parameters as packet transmission rate and average delay obtained. The novelty of the model is that the network metric is a function of time, and takes into account that the average queue length and the average delay take their limit values after convergence time.**

Keywords - **multipath routing, quality of service, flow-based model, tensor model.**

I. Introduction

Nowadays development of telecommunication networks (TCN) is characterized by the introduction of increasing number infocommunication services and associated technological means of improving the Quality of Service (QoS) of end-users. An important place in the list of such methods is given to QoS-routing over multiple parameters. Thus through informed choice of the routes for packets transmission between a pair of source and destination nodes determined numerical values of end-to-end QoS-parameters: average delay, jitter, packet loss probability and performance of TCN as a whole. However, existing technological and routing solutions based mostly on insufficient mathematical models and methods (graph models, method of finding the shortest path), and do not take into account the characteristics of packet flows making it difficult to control and prevent the overload of communication links.

Besides, effective solutions must be found for the problems of buffer resource management in TCN (queues of packets organized on the router interface), due to the fact that queue length changing contributes to packet delay variation (jitter). In this context, the technological mechanisms of queue management should have effective procedures to analyze the state of the router interface that enable management decisions in comply with the requirements for ensuring the specified level of QoS.

It was shown by the analysis [1,2], that it is essential to have models describing the dynamics of the network router interface state changes in time in order to obtain more accurate estimates of the queue length and associated QoS parameters. In this research proposed the dynamic presentation of tensor model for multipath routing with QoS guarantees over multiple parameters and different distribution laws of packet flows service time on the network router interface.

Oleksandr Lemeshko, Oleksandra Yeremenko - Kharkiv National University of Radio Electronics, Nauky Ave., 14, Kharkiv, 61166, UKRAINE,
E-mail: oleksandr.lemeshko@nure.ua; oleksandra.yeremenko@nure.ua

II. Flow-Based Model of Multipath Routing

Within the multipath routing model structure of TCN described by one-dimensional network $S=(U,V)$, where $U=\{u_i, i=\overline{1,m}\}$ is a set of network nodes (routers), and $V=\{v_z=(i,j); z=\overline{1,n}; i,j=\overline{1,m}; i\neq j\}$ is a set of edges (arcs). Here the edge $v_z=(i,j)\in V$ models z-th link connecting i-th and j-th routers. Assume that capacity $\phi_{(i,j)}$ is known for every link (i,j) measured as number of packets per second.

The result of routing problem solving is calculation of the set of routing variables $x^k_{(i,j)}$, each of which characterizes the fraction of intensity of the k-th flow directed from the node i to the node j via an appropriate j-th interface.

For the purpose of TCN nodes overload prevention it is necessary to meet the condition of flow conservation on the source, transit and destination nodes, respectively, which can be written in the form [3]:

$$\begin{cases} \sum\limits_{j:(i,j)\in V} x^k_{(i,j)} = 1, \ k\in K, \ i=s_k; \\ \sum\limits_{j:(i,j)\in V} x^k_{(i,j)} - \sum\limits_{j:(j,i)\in V} x^k_{(j,i)} = 0, \ k\in K, \ i\neq s_k, d_k; \\ \sum\limits_{j:(j,i)\in V} x^k_{(j,i)} = -1, \ k\in K, \ i=d_k, \end{cases} \quad (1)$$

where K is set of flows in the network; s_k is the source node and d_k is destination node for the k-th flow.

In order to implement multipath routing strategy with load balancing the control (routing) variables $x^k_{(i,j)}$ must satisfy the condition

$$0 \le x^k_{(i,j)} \le 1. \quad (2)$$

The precondition for controllability of routing is capacity constraint, i.e. the condition $\rho<1$ (where $\rho=\dfrac{\lambda}{\phi}$ is utilization of the link), therefore the model must contain the following expression:

$$\sum_{k\in K}\lambda_k^{\langle req\rangle}x_{(i,j)}^k < \phi_{(i,j)}, \quad (i,j)\in E, \tag{3}$$

where $\lambda_k^{\langle req\rangle}$ is average intensity of the k-th flow arriving into the network. This value is required packet transmission rate, one of QoS-metrics.

Because of the random nature of the network traffic performance of constraints (3) is only a necessary, but not sufficient condition for the absence of packet loss on the nodes of TCS caused by possible buffer overflow.

III. Dynamic Model of Queue Utilization on Network Router Interface

There are currently known a lot of types of mathematical models, based on different approximations of the dynamics of changes in state of TCN router interface. The most efficient in relation to adequacy and clarity, in our opinion, is a model based on the use of system of nonlinear differential equations of the network state $\dot{x}(t)=dx(t)/dt$ obtained by the Pointwise Stationary Fluid Flow Approximation (PSFFA) [1], where under the network state was understood the average queue length on the router interface. Using this model it is possible to estimate the influence of interface state, flow characteristics and packets service disciplines on the dynamics of router utilization. According to PSFFA, the flow conservation on the router interface and method of Pointwise Stationary Approximation, PSA, are combined into single nonlinear differential equation for the purpose of approximation of the non-stationary models at each time interval.

Within the chosen model there are known the following parameters: λ is the ensemble average flow rate (packets per second, 1/s) entering the analyzed queue; μ is the interface throughput (packets per second, 1/s) allocated to this queue. In modeling of the interface by queuing system M/G/1 (single-channel queuing system with Poisson arrival and general distribution of service time) dynamics of average queue length of the TCN router can be described by nonlinear differential equation of the form:

$$\dot{x}(t) = -\mu\left[\frac{x+1-\sqrt{x^2+2C_s^2+1}}{1-C_s^2}\right]+\lambda(t), \tag{4}$$

where C_s^2 is the squared coefficient of variation of the service time distribution.

Depending on the value of the coefficient C_s^2 may be determined the next special cases of model PSFFA M/G/1: M/M/1 (for $C_s^2=1$), M/D/1 (for $C_s^2=0$), and M/E$_k$/1 (for $C_s^2=1/k$, $k\geq 1$). Thus while using in approximation different types of queuing systems (4) will take form:

M/M/1:

$$\dot{x}(t) = -\mu\left(\frac{x}{x+1}\right)+\lambda, \tag{5}$$

M/D/1:

$$\dot{x}(t) = -\mu\left[(x+1)-\sqrt{x^2+1}\right]+\lambda, \tag{6}$$

M/E$_k$/1:

$$\dot{x}(t) = -\mu\left[\frac{k(x+1)}{k-1}-\frac{\sqrt{k^2x^2+2kx+k^2}}{k-1}\right]+\lambda, \tag{7}$$

where parameter k denotes the number of service stages.

IV. Tensor Model of Multipath QoS-Routing

In order to obtain a tensor model of the TCN let us introduce anisotropic space-structure constructed by the set of closed and open paths (circuits and node pairs). The dimension of this space is determined by the total number of edges in the network, and is equal to n [4]. Each independent path defines coordinate axis in the space-structure.

Assume S is a connected network, i.e. contains one connected component, then the cyclomatic number $\mu(S)$ and rank $\varphi(S)$ of the network define the number of basic circuits and node pairs, determining the validity of the following expressions:

$$\varphi(S)=m-1,\ \mu(S)=n-m+1,\ n=\varphi(S)+\mu(S). \tag{8}$$

From all possible circuits let us choose $\mu(S)$ of linearly independent circuits $\{\pi_i, i=\overline{1,\mu}\}$; set of all node pairs we will note as $\{\eta_j, j=\overline{1,\varphi}\}$. These sets form basis for n-dimensional space, corresponding to the structure of the network.

In introduced n-dimensional space the TCN can be described by mixed divalent tensor [5]

$$Q=T\otimes\Lambda, \tag{9}$$

where $\otimes$ is operator of direct tensor multiplication, and components of tensor Q are univalent covariant tensor of average packet delay T and univalent contravariant tensor of traffic intensity Λ.

Then, the expression (9) can be written in index form

$$q_j^i=\tau_j\lambda^i,\ i,j=\overline{1,n}, \tag{10}$$

where τ_j is packet transmission delay along the j-th coordinate path (s); λ^i is packet intensity of the flow transmitted along the i-th coordinate path (1/s).

Considering tensor (9) we will take into account the following two coordinate systems (CS): coordinate system of the edges (type v) and the coordinate system of the circuits and node pairs (type $\pi\eta$). Within tensor approach one of key role belongs to the variance of used tensors. The variance of tensor is related to rules of transformation of its coordinates in transition from one

CS to another. The variance of components of the tensor (10) is proved in [5,6]. It is known that the characteristics (metrics) which are submitting to conservation law, for example, the flow intensity, are contravariant values, and the additive parameters, for example, the average packet delay, are covariant values.

Consider an example, when the network router interface modeled by queuing system M/G/1 with special cases M/M/1, M/D/1, and M/E$_k$/1. Thus, according to approximations (5)-(7) and using Little`s law can be obtained the average packet delay on the network router interface.

For example, for M/M/1 we have

$$\dot{\tau}(t) = 1 - \mu\left(\frac{x}{\lambda x + 1}\right). \qquad (11)$$

Analytically obtained solution of differential equation (11) using MatLab environment is next:

$$\tau(t) = \frac{1}{\mu - \lambda}\cdot[(\mu \cdot W(0, -(\lambda \cdot \exp(-(\lambda + (t - -(\lambda + \mu \cdot \ln(\exp(-(\lambda \cdot (K\lambda - K\mu + 1))/\mu) \cdot \cdot (K\lambda - K\mu + 1)))/(\mu - \lambda)^2)\cdot(\mu - \lambda)^2)/\mu))/\mu))/ /\lambda + 1], \qquad (12)$$

where $W(\cdot)$ is Lambert W function; $\exp(\cdot)$ is exponential function; K is the average delay at the interface at initial time. For further tensor generalization the expression (9) can be written in the form

$$\tau_i^v(t) = \frac{1}{\lambda_v^i(\mu - \lambda)}\cdot[(\mu \cdot W(0, -(\lambda \cdot \exp(-(\lambda + (t - -(\lambda + \mu \cdot \ln(\exp(-(\lambda \cdot (K\lambda - K\mu + 1))/\mu) \cdot \cdot (K\lambda - K\mu + 1)))/(\mu - \lambda)^2)\cdot(\mu - \lambda)^2)/\mu))/\mu))/ /\lambda + 1]\lambda_v^i, \qquad (13)$$

where $i = \overline{1,n}$ shows number of link in TCN, but index v shows type of coordinate system (all parameters in (13) belong to edges where every edge models some link). In accordance with the Kron's postulate of the second generalizations [4], a system of equations (13) can be replaced by the following vector equation

$$\Lambda_v(t) = G_v(t)T_v(t), \qquad (14)$$

where $\Lambda_v(t)$ and $T_v(t)$ are projections of tensor $\Lambda(t)$ and tensor $T(t)$ in CS of edges respectively, represented as n-dimensional vectors of flow intensity and average packet delay in edges of the network; $G_v(t) = \|g_v^{ij}(t)\|$ is $n \times n$ diagonal matrix, diagonal elements of which are calculated according to (14), referred to the appropriate edges $\{v_i, i = \overline{1,n}\}$, i.e.

$$g_v^{ij}(t) = \lambda_v^i(\mu - \lambda)\cdot[(\mu \cdot W(0, -(\lambda \cdot \exp(-(\lambda + (t - -(\lambda + \mu \cdot \ln(\exp(-(\lambda \cdot (K\lambda - K\mu + 1))/\mu) \cdot \cdot (K\lambda - K\mu + 1)))/(\mu - \lambda)^2)\cdot(\mu - \lambda)^2)/\mu))/\mu))/ /\lambda + 1]^{-1}. \qquad (15)$$

According to definition [4,5] the coordinate transformation rules for tensors are linear and can be formalized by a nonsingular square $n \times n$ matrix C:

$$\Lambda_v(t) = C\,\Lambda_{\pi\eta}(t), \qquad (16)$$

where $\Lambda_{\pi\eta}(t)$ is n-dimensional vectors that are projection of tensor $\Lambda(t)$ in coordinate system of circuits and node pairs (type $\pi\eta$). In turn, the vector $\Lambda_{\pi\eta}(t)$ has components

$$\Lambda_{\pi\eta}(t) = \begin{bmatrix} \Lambda_\pi(t) \\ -- \\ \Lambda_\eta(t) \end{bmatrix} \; \Lambda_\pi(t) = \begin{bmatrix} \lambda_\pi^1(t) \\ \vdots \\ \lambda_\pi^j(t) \\ \vdots \\ \lambda_\pi^\mu(t) \end{bmatrix}; \Lambda_\eta(t) = \begin{bmatrix} \lambda_\eta^1(t) \\ \vdots \\ \lambda_\eta^p(t) \\ \vdots \\ \lambda_\eta^\varphi(t) \end{bmatrix} \qquad (17)$$

where $\Lambda_\pi(t)$ is μ-dimensional subvector related to flow intensities into circuits of the network; $\Lambda_\eta(t)$ is φ-dimensional subvector related to flow intensities between node pairs of the network; $\lambda_\pi^j(t)$ is the flow intensity in circuit π_j of the network; $\lambda_\eta^p(t)$ is the traffic intensity which is entering into the network and outgoing the network through the node pair η_p.

Projection of the tensor of average delays $T(t)$ in a coordinate system of circuits and node pairs (type $\pi\eta$) is represented by the n-dimensional vector with the following structure:

$$T_{\pi\eta}(t) = \begin{bmatrix} T_\pi(t) \\ -- \\ T_\eta(t) \end{bmatrix}; T_\pi(t) = \begin{bmatrix} \tau_1^\pi(t) \\ \vdots \\ \tau_j^\pi(t) \\ \vdots \\ \tau_\mu^\pi(t) \end{bmatrix}; T_\eta(t) = \begin{bmatrix} \tau_1^\eta(t) \\ \vdots \\ \tau_p^\eta(t) \\ \vdots \\ \tau_\varphi^\eta(t) \end{bmatrix}, \qquad (18)$$

where $\tau_j^\pi(t)$, $\tau_p^\eta(t)$ are average packet delay in circuit π_j and between node pair η_p respectively. Sizes of $T_\pi(t)$ and $T_\eta(t)$ are μ and φ respectively.

Covariant character of delay tensor $T(t)$ causes following law of the coordinate transformation:

$$T_v(t) = A\ T_{\pi\eta}(t), \qquad (19)$$

where A is $n \times n$ matrix of covariant coordinate transformation in transition from CS of circuits and node pairs to CS of edges.

The matrix A is related to matrix C (matrix of contravariant coordinate transformation) by orthogonality condition $CA^t = I$; I is unity $n \times n$ matrix, $[\cdot]^t$ is transposition operator.

Expression (14) has the same form in coordinate system of circuits and node pair:

$$\Lambda_{\pi\eta}(t) = G_{\pi\eta}(t)\,T_{\pi\eta}(t). \qquad (20)$$

Then $G(t)$ is divalent contravariant metric tensor:

$$G_{\pi\eta}(t) = A^{t} G_{v}(t) A, \qquad (21)$$

where $G_{\pi\eta}(t)$ is projection of tensor $G(t)$ in CS of circuits and node pair.

Let us consider within the tensor model (8)-(21) the problem of conditions for QoS ensuring in TCN. Assume we have numerical requirements of average delay $\tau_{\langle req\rangle}$ and packet transmission rate $\lambda^{\langle req\rangle}$. Our goal is obtaining analytical conditions for ensuring all of the requirements. Let us agree that node pair source-destination will have the first number (17). Then the required condition for QoS ensuring are [5-7]

$$\lambda^{\langle req\rangle} \le \left(G_{\pi\eta}^{\langle 4,1\rangle} - G_{\pi\eta}^{\langle 4,2\rangle} \left[G_{\pi\eta}^{\langle 4,4\rangle} \right]^{-1} G_{\pi\eta}^{\langle 4,3\rangle} \right) \tau_{\langle req\rangle}, \quad (22)$$

where

$$G_{\pi\eta} = \left\| \begin{array}{ccc} G_{\pi\eta}^{\langle 1\rangle} & | & G_{\pi\eta}^{\langle 2\rangle} \\ --- & + & --- \\ G_{\pi\eta}^{\langle 3\rangle} & | & G_{\pi\eta}^{\langle 4\rangle} \end{array} \right\|; G_{\pi\eta}^{\langle 4\rangle} = \left\| \begin{array}{ccc} G_{\pi\eta}^{\langle 4,1\rangle} & | & G_{\pi\eta}^{\langle 4,2\rangle} \\ --- & + & --- \\ G_{\pi\eta}^{\langle 4,3\rangle} & | & G_{\pi\eta}^{\langle 4,4\rangle} \end{array} \right\|,$$

and $G_{\pi\eta}^{\langle 1\rangle}$ are square $\varphi \times \varphi$ and $\mu \times \mu$ submatrices, respectively, $G_{\pi\eta}^{\langle 2\rangle}$ is $\mu \times \varphi$ submatrix, $G_{\pi\eta}^{\langle 3\rangle}$ is $\varphi \times \mu$ submatrix; $G_{\pi\eta}^{\langle 4,1\rangle}$ is the first element of the matrix $G_{\pi\eta}^{\langle 4\rangle}$, and according to (15) and (21) all their coordinates are functions of time.

Inequality (20) is desired condition to ensure QoS on set of heterogeneous metrics (transmission rate $\lambda^{\langle req\rangle}$ and time $\tau_{\langle req\rangle}$) simultaneously. In addition, the derivation of the inequality (22) is based on requirement $T_{\pi} = 0$. It means delay in every circuit must be zero, and it guarantees that delay in pair source-destination will be minimal. Delays along different paths will be same and jitter caused by multipath routing minimized.

V. Conclusion

In this research was obtained the new dynamic presentation of tensor model for multipath routing with QoS guarantees over multiple parameters. The model is based on the flow-based conservation law (1). The expressions for different queuing systems (5)-(7) modeling state of the network router interface used for representing the average packet delay as time-varying function. Conditions of ensuring QoS (22) for the set of parameters as packet transmission rate and average delay obtained in the same way as in [4]. The novelty of the model is that the network metric is a function of time (15), i.e. it takes into account that the average queue length (5)-(7) and the average delay take their limit values are not instantly, but after some time (convergence). As was shown by simulation, this time is within the range of few to tens of seconds. Therefore, given that the routing timers are from 30 up to 90 seconds, disregard the convergence time is not acceptable. Thus, using model (1)-(22) allows more adequate describing the routing process in terms of packet transmission rate and average delay.

Proposed model can be used in calculation of important network parameters (average packet delay, network utilization), and can be the basis for a new QoS-routing protocols in modern multiservice networks.

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