

ABOUT ONE PROBLEM OF OPTIMAL STOCHASTIC CONTROL OF THE MODES OF OPERATION OF WATER MAINS

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Abstract: The problem of increasing of the efficiency of operation of the water mains in modern conditions while the transition to a three-tier tariff for the electricity is examined in the present work. An effective method for solving this problem, based on the use of specific features of the water mains as stochastic objects operating in the stochastic environment is offered. The mathematical formulation of the problems of optimal stochastic control of the modes of operation of the water main with probabilistic constraints on the phase variables is presented. A new strategy for the optimal stochastic control of the modes of operation of the water main, the use of which has allowed to develop an effective method for solving the examined problem is proposed in the present work. It is shown that the transition from the classical deterministic problems of control of the modes of operation of the water mains to stochastic ones, provides a significant (up 9%) decrease of financial expenses for the electricity.

Keywords: optimal stochastic control, probabilistic constraints on the phase variables, water main, pumping station.

INTRODUCTION

A water main (WM) is a complex technical system designed to transport water over long distances. WM consists of the sequence of multishop pumping stations (PS) and multiline main pipelines. [1] There are clean-water reservoirs (CWR) at the entrance of each PS. In CWR of the first PS of WM prepared water is supplied from one or several ascents. At the exit of WM, as a rule, there are CWR of large capacity used as the sources of water supply for cities and towns [10, 11, 12].

The main controlled elements of WM are pump units (PU) of PS. Each shop of PS is represented by several connected in parallel PU. Control of the operation of PU is carried out by switching on / off PU of PS, by changing the position of the adjustable valve (AV), by adjustable speed control of the drive motors of PU. Control of the operation of WM is carried out by changing the modes of operation of PS [5, 10, 13].

The actuality of the problem. Spur increase of electricity tariffs and the introduction of a three-tier tariff created necessary conditions for the transition to energy-saving technologies of WM control.

The purpose of this work is the development of a method of optimal stochastic control of the modes of operation of WM, the implementation of which provides a significant reduction in financial expenses for the electricity.

To develop more adequate mathematical models of the modes of operation of WM and more effective account of the specific features of WM as an object of

control, WM is considered as a stochastic object operating in the stochastic environment. The presence in WM of CWR of sufficiently large capacity is such specific feature. The use of this feature of WM allows to build the strategy for optimization of the modes of its operation, based on the maximum possible use of the whole capacity of the reservoirs. The main point of this strategy is that the water supply into CWR from WM must be minimal at the time interval with the maximum rate and maximum at the time interval with the minimum rate. Using of such a strategy leads to the need in introduction of additional extreme constraints on the phase variables (water levels in CWR): by the time of the transition of the tariff for the electricity from smaller to larger - the mathematical expectations of water levels in CWR should be maximum possible, and before the start time of the minimum rate, the mathematical expectations of water levels in CWR should be minimal. Moreover, to prevent the occurrence of accidents at any time intervals $t \in [0, T]$ the probability of overflow or emptying of the reservoirs must be close to zero.

The application of this strategy has led to the need to develop problems of optimal stochastic control of the modes of operation of WM with extreme and probabilistic constraints on the phase variables of a new class.

THE MATHEMATICAL FORMULATION OF THE PROBLEM

The main criterion of the effectiveness of the operation of WM is the value of the mathematical expectation of the total cost of the electricity consumed by all PS at a predetermined interval of operation $[0, T]$, on condition that WM provides water to all consumers in the required (predicted) volumes.

For the formulation of the problem of optimal stochastic control of the modes of operation of WM we will use the stochastic model of quasi-stationary modes of operation of water supply and water distribution systems [19 - 23].

Hydraulic resistances of the sections of water main are unknown a priori, so they are estimated according to the experimental data. As the evaluation is carried out on samples of the experimental data of the final length, these estimations are random variables, statistical properties of which depend both on the statistical properties of the errors of flow measurement q and water head h , and the estimation method and sample size. It is supposed that the errors of measurement of the variables h and q are normally distributed random variables. As the method of estimating, the maximum probability method is used, so the obtained estimates of random variables $S_i(q_{ik}(\omega))$,

($i=1, \dots, v+\eta_2+\xi_1-1$) are unbiased, efficient, having normal distribution.

The initial data for the problem of optimal stochastic control of the modes of operation of WM are the interval of control $[0, T]$ (one day), which is divided into 24 subintervals corresponding to each hour of the period of control $k=0, \dots, 23$. At each k subinterval of the time the predictions of consumption of all consumers of WM are known in the form of conditional mathematical expectations $\bar{q}_{i0}(l) = M_{\omega}(q_{il}(\omega))$ and their variances $\sigma_{q_{i0}}^2(l) = D_{\omega}(q_{il}(\omega))$, calculated at the time interval $k=0$ proactively $l=1, 2, \dots, 23$; the prediction of daily water consumption from CWR $\bar{q}_{z\text{vihl}}(l) = M_{\omega}(q_{z\text{vihl}}(\omega))$, the measured values of the water level in the z CWR $H_{zk}(\tilde{\omega})$; the actual number of switched PU, and static data: the structure of WM, lengths, diameters, geodesic marks of the sections of the pipeline, the estimates of the parameters of the model of PU on each PS, the physical size of CWR, the estimates of hydraulic resistance of AV on each PS.

The objective function of the problem of optimal stochastic control of the modes of operation of WM per day we can present as the mathematical expectation of the total value of the electricity consumed by all PS of WM at the interval of control $[0, T]$:

$$M_{\omega} \sum_{k=0}^{23} \sum_{i=1}^n \sum_{j=1}^m N_{ijk}(q_{ik}(\omega)) \cdot r_k \rightarrow \min_{u(k) \in \Omega}. \quad (1)$$

The area of restriction Ω is determined by the stochastic model of quasi-stationary modes of water supply [7, 8, 23]:

$$M_{\omega} \left(h_{rk}(q_{rk}(\omega)) + \sum_{i \in L} b_{1ri} h_{NAik}(q_{ik}(\omega)) + \sum_{i \in R} b_{1ri} h_{RZik}(q_{ik}(\omega)) + \sum_{i \in M} b_{1ri} h_{ik}(q_{ik}(\omega)) \right) = 0, \quad (2)$$

$$(r = v, \dots, v+\eta_2-1; k = 0, \dots, 23),$$

$$M_{\omega} \left(h_{NSjk}(\omega) - H_{zk}(\omega) - h_{NAjrk}(q_{rk}(\omega)) + h_{RZjrk}(q_{rk}(\omega)) + \sum_{i \in M} b_{1ri} (h_{ik}(q_{ik}(\omega)) + h_i^{(g)}) \right) = 0, \quad (3)$$

$$(j = 1, \dots, n; r = 1, \dots, m; z = 1, \dots, Z).$$

$$q_{ik}(\omega) = M_{\omega} \left(\sum_{r=v}^{v+\eta_2-1} b_{1ri} q_{rk}(\omega) + \sum_{r=v+\eta_2}^e b_{1ri} q_{rk}(\omega) \right), \quad (4)$$

$$(i = 1, \dots, v-1),$$

$$q_{ik}(\omega) > 0, \quad i \in L. \quad (5)$$

$$h_{ik}(q_{ik}(\omega)) = \text{sgn } q_{ik}(\omega) S_i(\omega) q_{ik}^2(\omega), \quad i \in M, \quad (6)$$

$$h_{NAik}(q_{ik}(\omega)) = a_{0i}(\omega) + a_{1i}(\omega) q_{ik}(\omega) + a_{2i}(\omega) q_{ik}^2(\omega), \quad i \in L, \quad (7)$$

$$\eta_{NAik}(q_{ik}(\omega)) = d_{0i}(\omega) + d_{1i}(\omega) q_{ik}(\omega) + d_{2i}(\omega) q_{ik}^2(\omega), \quad i \in L, \quad (8)$$

$$N_{NAik}(q_{ik}(\omega)) = \frac{9,81 \cdot h_{NAik}(q_{ik}(\omega)) \cdot q_{ik}(\omega)}{0,9 \cdot \eta_{NAik}(q_{ik}(\omega))}, \quad i \in L, \quad (9)$$

$$h_{RZik}(q_{ik}(\omega)) = \frac{q_{ik}(\omega) C_i(\omega)}{E_{ik}^2}, \quad i \in R, \quad (10)$$

and the models of reservoirs

$$H_{zk}(\omega) = H_{z,k-1}(\omega) + c_{zk}(q_{z\text{v}hk}(\omega) - q_{z\text{v}ihk}(\omega)), \quad (z = 1, \dots, Z), \quad (11)$$

with probabilistic constraints on the phase variables:

$$P(H_{zk}(\omega) \leq H_z^{\max}) \geq \alpha, \quad (12)$$

$$P(H_{zk}(\omega) \geq H_z^{\min}) \geq \alpha,$$

$$\alpha \approx 0,97,$$

and extreme values of constrains for fixed time intervals $k=6$ and $k=23$:

$$M_{\omega} \{H_{z6}(\omega)\} \rightarrow \max_{q_{z\text{v}hk} \in \Omega}, \quad (k=0, \dots, 6), \quad (13)$$

$$M_{\omega} \{H_{z23}(\omega)\} \rightarrow \min_{q_{z\text{v}hk} \in \Omega}, \quad (k=0, \dots, 23), \quad (14)$$

where $u(k)$ - vector of control which determines the amount of operating PU, the position of AV; $H_{zk}(\omega)$ - water level in z CWR at a given k time interval, H_z^{\min} - minimum allowable water level in z CWR, H_z^{\max} - maximum allowable water level in z CWR.

Random variables characterize: $q_{ik}(\omega)$ - water consumption on i section of the pipeline at k time interval; $h_{NSjk}(\omega)$ - water head at the output of PS, $h_{NAik}(q_{ik}(\omega))$ - water head of i PU. $S_i(\omega)$ - evaluation of the hydraulic resistance of i section of the pipeline ($i \in M$); $h_{RZik}(q_{ik}(\omega))$ - evaluation of the head fall on i AV; $\eta_{NAik}(q_{ik}(\omega))$ - evaluation of the coefficient of efficiency of i PU; $a_{0i}(\omega), a_{1i}(\omega), a_{2i}(\omega), d_{0i}(\omega), d_{1i}(\omega), d_{2i}(\omega)$ - evaluations of the parameters of PU ($i \in L$); $C_i(\omega)$ - evaluations of the parameters of AV ($i \in R$); E_{ik} - rate of the opening of AV ($E \in (0,1]$); $h_i^{(g)}$ - geodesic mark of i section of the pipeline ($i \in M$), b_{1ri} - cyclomatic matrix element; $N_{NAik}(q_{ik}(\omega))$ - power rating of PU; n - number of PS; m - number of PU on the selected PS; $M_{\omega}\{\cdot\}$ - mathematical expectation of the random variable $\{\cdot\}$.

For the solvability of the problem (1) - (14) for $k=0$ the predictions of the expenses as conditional mathematical expectations, calculated at time interval $k=0$, proactively $l=1, 2, \dots, 23$ all incidental consumers $q_{ik}(l)$, receiving water from WM and final consumers $q_{jk}(l)$, receiving water from the reservoir at the output of WM must be known additionally; water levels in each z CWR - H_{z0} .

The problem of optimal stochastic control of the modes of operation of WM (1) - (14) belongs to the class of nonlinear problems of optimal stochastic control with discrete time [2, 3, 4, 9, 17, 18], and probabilistic constraints on the phase variables. There aren't any exact solutions of the problems of such a class nowadays. The approximate method of solving the examined problem by the transition from stochastic problem (1) - (14) to its

deterministic equivalent the decision of which is carried out by the modified method of branches and bounds is presented in this work.

The cost of the electricity is determined by a three-tier tariff shown in Table 1.

Table 1. The electricity rate according to the hours

Hours	6.00-8.00	8.00-10.00	10.00-18.00	18.00-22.00	22.00-23.00	23.00-6.00
Coefficient	1,02	1,68	1,02	1,68	1,02	0,35

The deterministic equivalent of the problem of optimal stochastic control of the modes of operation of WM at the time interval $[0, T]$ takes the form:

$$\sum_{k=1}^{24} \sum_{i=1}^n \sum_{j=1}^{m_i} \bar{N}_{ijk}(\bar{q}_{ik}) \cdot r_k \rightarrow \min_{u(k)}, k = 0, \dots, 23, \quad (15)$$

$$\begin{aligned} & \bar{h}_{rk}(\bar{q}_{rk}) + \sum_{i \in L} b_{1ri} \bar{h}_{NAik}(\bar{q}_{ik}) + \sum_{i \in R} b_{1ri} \bar{h}_{RZik}(\bar{q}_{ik}) + \\ & + \sum_{i \in M} b_{1ri} \bar{h}_{ik}(\bar{q}_{ik}) = 0, \end{aligned} \quad (16)$$

$$\begin{aligned} & (r = v, \dots, v + \eta_2 - 1), \\ & \bar{h}_{NSjk} - \bar{H}_{zk} - \bar{h}_{NAjrk}(\bar{q}_{rk}) + \bar{h}_{RZjrk}(\bar{q}_{rk}) + \\ & + \sum_{i \in M} b_{1ri} (\bar{h}_{ik}(\bar{q}_{ik}) + h_i^{(g)}) = 0, \\ & (j = 1, \dots, n; \dots r = 1, \dots, m). \end{aligned} \quad (17)$$

$$\bar{q}_{ik} = \sum_{r=v}^{v+\eta_2-1} b_{1ri} \bar{q}_{rk} + \sum_{r=v+\eta_2}^e b_{1ri} \bar{q}_{rk}, (i = 1, \dots, v-1), \quad (18)$$

$$\bar{q}_{ik} > 0, \quad i \in L. \quad (19)$$

$$\bar{h}_{NAik}(\bar{q}_{ik}) = \bar{a}_{0i} + \bar{a}_{1i} \bar{q}_{ik} + \bar{a}_{2i} \bar{q}_{ik}^2, \quad i \in L, \quad (20)$$

$$\bar{\eta}_{NAik}(\bar{q}_{ik}) = \bar{d}_{0i} + \bar{d}_{1i} \bar{q}_{ik} + \bar{d}_{2i} \bar{q}_{ik}^2, \quad i \in L, \quad (21)$$

$$\bar{N}_{NAik}(\bar{q}_{ik}) = \frac{9,81 \cdot \bar{h}_{NAik}(\bar{q}_{ik}) \cdot \bar{q}_{ik}}{0,9 \cdot \bar{\eta}_{NAik}(\bar{q}_{ik})}, \quad i \in L, \quad (22)$$

$$\bar{h}_{RZik}(\bar{q}_{ik}) = \frac{\bar{q}_{ik} \bar{C}_i}{E_{ik}^2}, \quad i \in R, \quad (23)$$

$$\bar{h}_{ik}(\bar{q}_{ik}) = \text{sgn} \bar{q}_{ik} \bar{S}_i \bar{q}_{ik}^2, \quad i \in M, \quad (24)$$

$$\bar{H}_{zk} = \bar{H}_{zk-1} + c_{zk} (\bar{q}_{zvhk} - \bar{q}_{zvihk}), \quad (z = 1, \dots, Z), \quad (25)$$

$$H_z^{-*} \leq \bar{H}_{zk} \leq H_z^{+*}, \quad (k=1,2,\dots,23), \quad (26)$$

$$\bar{H}_{z6} \rightarrow \max_{q_{zvhk} \in \Omega}, \quad (k=0,\dots,6), \quad (27)$$

$$\bar{H}_{z23} \rightarrow \min_{q_{zvhk} \in \Omega}, \quad (k=0,\dots,23), \quad (28)$$

H_z^{-*}, H_z^{+*} - calculated values of the minimum and maximum water levels in z CWR, where for $\forall \omega \in \Omega$ probabilistic constraints will be fulfilled (12).

As a result of the solving the problem (15) - (28) of optimal stochastic control of the modes of operation of WM for each time interval k , we obtain:

1. vector of control $u(k)$ which, for each time interval k includes: the number of operating PU, the position of the operating point of each PU;
2. estimates of the mathematical expectations of water levels in all CWR;
3. estimates of the mathematical expectations of the expenses and pressure drops in all sections of the pipeline;
4. estimates of the mathematical expectations of the expenses for the electricity and its value in accordance with the three-tier tariff, by all the operating PU on all the PS of WM.

At the time interval $k=23$, we obtain the estimates of the mathematical expectations of the total value of the electricity consumed by each PS at the interval $[0, T]$.

RESULTS AND DISCUSSION

The evaluation of the effectiveness of the proposed method was carried out for the WM, the structure of which is shown in Fig. 1.

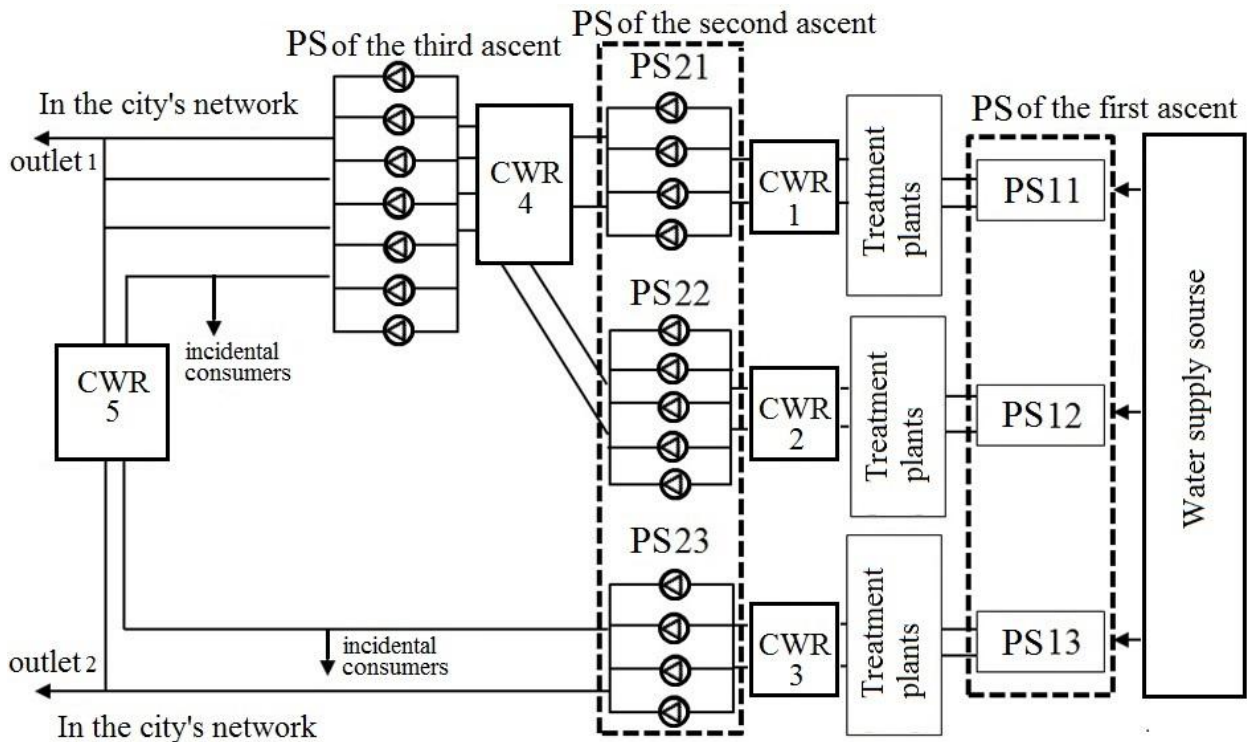


Fig.1. The structure of the water main

The examined WM includes three pumping stations of the first ascent (PS11, PS12, PS13), three PS of the second ascent (PS21, PS22, PS23) and PS of the third ascent. PS of the second ascent PS21 and PS22 supply water to the PS of the third ascent, and PS of the third ascent and PS23 supply water into CWR5 and into the water supply of a city.

To solve the problem of optimal stochastic control of the modes of operation of WM, PS of the second and third ascents and CWR4 and CWR5 have been examined.

The initial data for the problem of optimal stochastic control of the modes of operation of WM at the time interval $[0, T]$ are:

- static data including the structure of WM; lengths; diameters; geodetic marks of the sections of the pipeline; estimates of the parameters of the mathematical models of PU for each PS; estimates of the hydraulic resistances of AV on each PS; physical dimensions of each CWR;

- dynamic data, including the prediction of the daily water consumption from CWR5; the prediction of water consumption by incidental consumers of the city, bypassing CWR5.

PS of the second ascent PS21 and PS23 are equipped with four similar, connected in parallel PU with the same characteristics, PS22 and PS of the third ascent are equipped with five and seven similar PU with the different characteristics correspondingly. Approximated parameters of PU are shown in Table 2, passport characteristics of PU are in Fig. 2, Fig. 3. The net graph is shown in Fig. 4.

In Fig. 5 there is a diagram of the prediction of the process of hourly water consumption from CWR5 calculated at the time interval $k=8$ proactively $l=1, \dots, 23$ for each of seven days.

Table 2. The characteristics of the pumping units

Number of PU	a0	a1	a2	d0	d1	d2
PS1 – PU type 20 NDS (D3200-75)						
PU 1-4	85,0124	-0,15429	-15,7371	-0,14286	96,45429	-26,1514
PS2 – PU type 24 NDS (D6300-80)						
PU 1, 4	62,76	-1,872	-4,32	-0,14286	96,4543	-26,1514
PU 2, 3, 5	93,5006	-0,21695	-4,3193	-0,14286	96,4543	-26,1514
PS3 – PU type22 NDS (D4000-95)						
PU 1-4	109,08	-2,03524	-8,94861	0	147,0046	-61,0308
PS of the third ascent – PU type 24 NDS (D6300-80)						
PU2	87,32864	-0,21375	-4,25547	-0,14286	96,4543	-26,1514
PU3	80,14337	-0,21375	-4,25547	-0,14286	96,4543	-26,1514
PU4	90,73704	-0,21375	-4,25547	-0,14286	96,4543	-26,1514
PU5	93,5006	-0,21375	-4,25547	-0,14286	96,4543	-26,1514
PU6	83,2662	-0,21375	-4,25547	-0,14286	96,4543	-26,1514

PU7	93,5006	-0,21375	-4,25547	-0,14286	96,4543	-26,1514
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The H-Q characteristics of the pump units

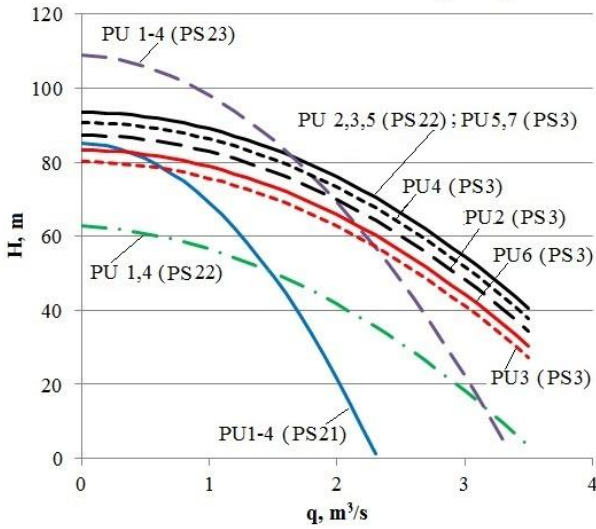


Fig. 2. The characteristics of the pump units

COP-Q characteristics of the pump units

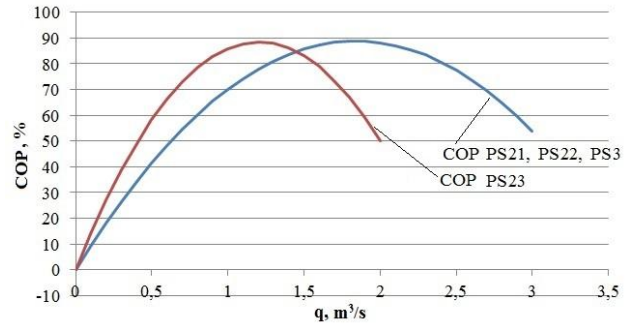


Fig. 3. COP-Q characteristics of the pump units

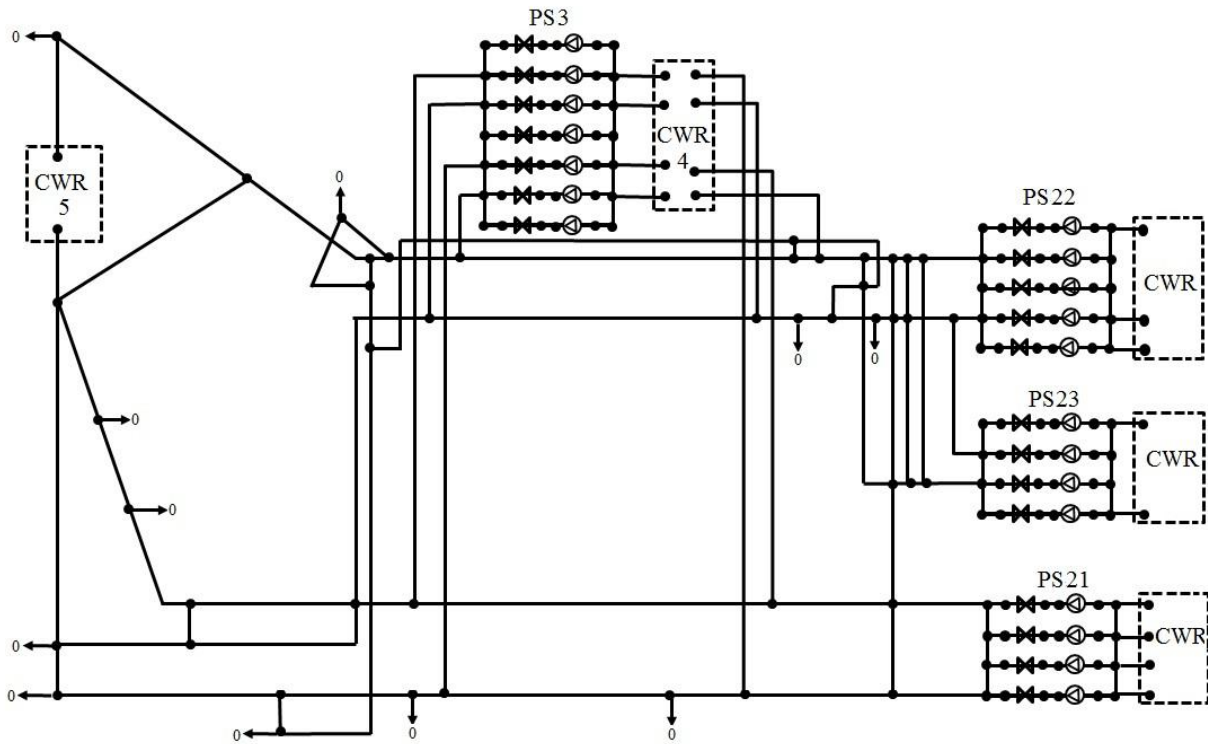


Fig.4. The net graph (● - vertex; ⊕ - pump unit; ⊗ - adjustable valve; — - arc; ↓ - final consumer)

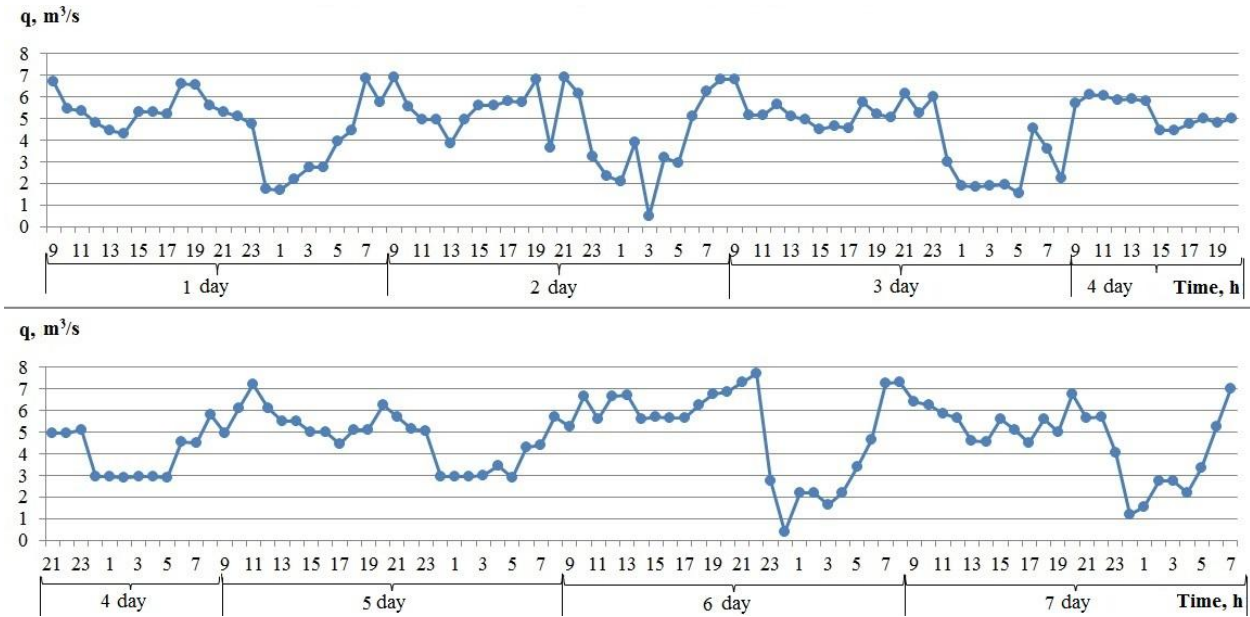


Fig. 5. The diagram of the prediction of hourly water consumption from CWR5 for seven days

At time zero $k = 0$ for the actual and optimal mode the same conditions have been used: mathematical expectation of water levels in CWR4 $H_{2,9} = 2,6$ m; in CWR5 $H_{1,9} = 3,9$ m; PU1 operated on PS21; PU3 and

PU5 operated on PS22; PU2 operated on PS23; PU3 and PU6 operated on PS3.

The results of solving the problem of optimal stochastic control of the modes of operation of WM are shown in Table 3.

Table 3. The numbers of operating pump units for the actual (a) and optimal (o) modes of operation of the water main

Hours	Tariff	PS21			PS22						PS23			
		a		o	a		o				a		o	
		Day												
		1-7	1,5	2-4, 6,7	1-5	6,7	1,5	2,3,6	4	7	1,2	3,4,5	6,7	1-7
9	1,68	1	1	1	3,5	3,4	3,5	3	3	3	2	2	2	2
10	1,68	1	1	1	3,5	3,4	3,5	3	3	3	2	2	2	2
11	1,68	1	1	1	3,5	3,4	3,5	3	3	3	2	2	2	2
12	1,02	1	1	1	3,5	3,4	3,5	3,5	3	3	2	2	2	2
13	1,02	1	1	1	3,5	3,4	3,5	3,5	3,5	3,5	2	2	2	2
14	1,02	1	1	1	3,5	3,4	3,5	3,5	3,5	3,5	2	2	2	2
15	1,02	1	1	1	3,5	3,4	3,5	3,5	3,5	3,5	2	2	2	2
16	1,02	1	1	1	3,5	3,4	3,5	3,5	3,5	3,5	2	2	2	2
17	1,02	1	1	1	3,5	3,4	3,5	3,5	3,5	3,5	2	2	2	2
18	1,02	1	1	1	3,5	3,4	3,5	3,5	3,5	3,5	2	2	2	2
19	1,02	1	1	1	3,5	3,4	3,5	3,5	3,5	3,5	2	2	2	2
20	1,02	1	1	1	3,5	3,4	3,5	3,5	3,5	3,5	2	2	2	2
21	1,68	1	1	1	3,5	3,4	3	3	3	3,5	2	2	2	2
22	1,68	1	1	1	3,5	3,4	3	3	3	3	2	2	2	2
23	1,68	1	1	1	3,5	3,4	3	3	3	3	2	2	2	2
24	0,35	1	1,2	1,2	3,5	3,4	3,5,2	3,5,2	3,5,2	3,5,2	2	2	2,3	2,3
1	0,35	1	1,2	1,2	3,5	3,4	3,5,2	3,5,2	3,5,2	3,5,2	2	2	2,3	2,3
2	0,35	1	1,2	1,2	3,5	3,4	3,5,2	3,5,2	3,5,2	3,5,2	2,3	2	2,3	2,3
3	0,35	1	1,2	1,2	3,5	3,4	3,5,2	3,5,2	3,5,2	3,5,2	2,3	2	2,3	2,3
4	0,35	1	1,2	1,2	3,5	3,4	3,5,2	3,5,2	3,5,2	3,5,2	2,3	2	2,3	2,3
5	0,35	1	1,2	1,2	3,5	3,4	3,5,2	3,5,2	3,5,2	3,5,2	2,3	2	2,3	2,3
6	0,35	1	1,2	1,2	3,5	3,4	3,5,2	3,5,2	3,5,2	3,5,2	2,3	2	2,3	2,3
7	1,02	1	1,2	1	3,5	3,4	3,5,2	3,5	3,5	3,5	2	2	2	2
8	1,02	1	1,2	1	3,5	3,4	3,5,2	3,5	3,5	3,5	2	2	2	2

Table 3 (Continue). The numbers of operating pump units for the actual (a) and optimal (o) modes of operation of the water main

Hours	Tariff	PS3													
		a							o						
		Day													
		1	2	3	4,5	6	7	1	2	3	4	5	6	7	
9	1,68	3,6	3,4	3,5	3,5	3,5	3,5	3	3	3	3	3	3	3	
10	1,68	3,6	3,4	3,5	3,5	3,5	3,5	3	3	3	3	3	3	3	
11	1,68	3,6	3,4	3,5	3,5	3,5	3,5	3	3	3	3	3	3	3	
12	1,02	3,6	3,4	3,5	3,5	3,5	3,5	3,6	3	3	3	3	3,5	3	
13	1,02	3,6	3,4	3,5	3,5	3,5	3,5	3,6	3	3	3	3	3,5	3	
14	1,02	3,6	3,4	3,5	3,5	3,5	3,5	3,6	3	3	3	3	3,5	3	
15	1,02	3,6	3,4	3,5	3,5	3,5	3,5	3,6	3	3	3	3	3,5	3	
16	1,02	3,6	3,4	3,5	3,5	3,5	3,5	3,6	3	3	3	3	3,5	3	
17	1,02	3,6	3,4	3,5	3,5	3,5	3,5	3,6	3	3	3	3	3,5	3	
18	1,02	3,6	3,4	3,5	3,5	3,5	3,5	3,6	3	3	3	3	3,5	3	
19	1,02	3,7	3,4	3,5	3,5	3,5	3,5	3,6	3	3	3	3	3,5	3,5	
20	1,02	3,7	3,4	3,5	3,5	3,5	3,5	3,6	3	3	3	3	3,5	3,5	
21	1,68	3,7	3,4	3,5	3,5	3,5	3,5	3	3	3	3	3	3	3	
22	1,68	3,7	3,4	3,5	3,5	3,5	3,5	3	3	3	3	3	3	3	
23	1,68	3,7	3,4	3,5	3,5	3,5	3,5	3	3	3	3	3	3	3	
24	0,35	3,7	3,4	3,5	3,5	3,5,7	3,5,7	3,6,7,5,4	3,4,5,7,2	3,4,5,7	3,5,6,7	3,5,6,7,4	3,5,6,7	3,5,6,7,4	
1	0,35	3,6,7	3,4,7	3,5,6	3,5	3,5,7	3,5,7	3,6,7,5,4	3,4,5,7,2	3,4,5,7	3,5,6,7	3,5,6,7,4	3,5,6,7	3,5,6,7,4	
2	0,35	3,6,7	3,4,7	3,5,6	3,5	3,5,7	3,5,7	3,6,7,5,4	3,4,5,7,2	3,4,5,7	3,5,6	3,5,6,7,4	3,5,6,7	3,5,6,7,4	
3	0,35	3,6,7	3,4,7	3,5,6	3,5	3,5,7	3,5,7	3,6,7,5,4	3,4,5,7,2	3,4,5	3,5,6	3,5,6,7,4	3,5,6,7	3,5,6,7,4	
4	0,35	3,6,7	3,4,7	3,5,6	3,5	3,5,7	3,5,7	3,6,7,5	3,4,5,7,2	3,4,5	3,5,6	3,5,6,7,4	3,5,6,7	3,5,6,7	
5	0,35	3,6,7	3,4,7	3,5,6	3,5	3,5,7	3,5,7	3,6,7,5,4	3,4,5,7,2	3,4,5	3,5,6	3,5,6,7	3,5,6,7	3,5,6,7	
6	0,35	3,6,7	3,4,7	3,5,6	3,5	3,5,7	3,5,7	3,6,7,5,4	3,4,5,7,2	3,4,5	3,5,6	3,5,6,7	3,5,6,7	3,5,6,7	
7	1,02	3,7	3,4	3,5	3,5	3,5	3,5,7	3,6	3	3	3	3	3,5	3	
8	1,02	3,7	3,4	3,5	3,5	3,5	3,5	3,6	3	3	3	3	3,5	3	

The use of the proposed strategy for solving the problem of optimal stochastic control of the modes of operation of WM has enabled to use all possible range of changes of the water level in CWR4 and CWR5 and use the technical characteristics of PU more efficiently which provides a significant (up 9%) savings of financial costs

for the electricity at the time interval $[0, T]$ and using a three-tier electricity tariff.

Table. 4 presents the estimates of the mathematical expectation of the power and cost value for the electricity of PS of the second and third ascents during seven days for the actual and optimal modes of operation of WM.

Table 4. The comparative analysis of the estimates of the mathematical expectation of the costs of the electricity and financial costs of the operation of pumping stations of the second and third ascents for the actual (a) and optimal (o) modes

Day	N, kW (a)	N, kW (o)	Cost, UAH (a)	Cost, UAH (o)
1	261725,6	259592,8	248795,2	258043,1
2	263471,0	239818,4	250998,3	216232,7
3	255218,4	241436,4	246817,1	209520,6
4	247644,9	243391,7	245269,2	209237,7
5	246623,4	238565,2	244069,6	219133,4
6	268072,1	265745,9	252127,2	254190,9
7	268385,5	266095,8	252291,3	220425,2
Amount	1811140,9	1754646,0	1740367,9	1586783,7

From Table 4 it can be seen that as a result of the transition of the mode of operation of WM from the actual to the optimal one during seven days, we managed to save 153584 UAH., which is 8,82% of the previous amount.

Fig. 6 shows the change of the estimate of the mathematical expectation of the cost value for the electrical energy while the transition from the actual to optimal mode.

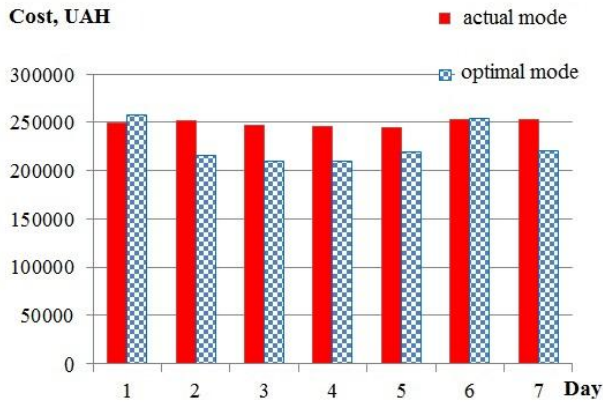


Fig. 6 The change of the estimate of the mathematical expectation of the value cost for the electrical energy consumed by the pumping station of the second and third ascents while the transition from the actual to the optimal mode.

In Tab. 5 the estimates of the target values of upper and lower bounds of water levels in CWR4 and CWR5 and also the bounds of the ranges of water level changes for the actual and optimal modes at the examined interval of seven days are shown.

Table 5. The change of water levels in CWR4 and CWR5

	Water level in CWR4, m		Water level in CWR5, m	
	min	max	min	max
Limitations	2,00	4,9	1,45	4,9
Actual mode	2,38	3,84	1,45	4,5
Optimal mode	2,6	4,83	1,81	4,89

Fig. 7, Fig. 8 show the change of the estimate of the mathematical expectation of water levels in CWR4 and CWR5 as a result of actual and optimal mode of operation during seven days of the planned period.

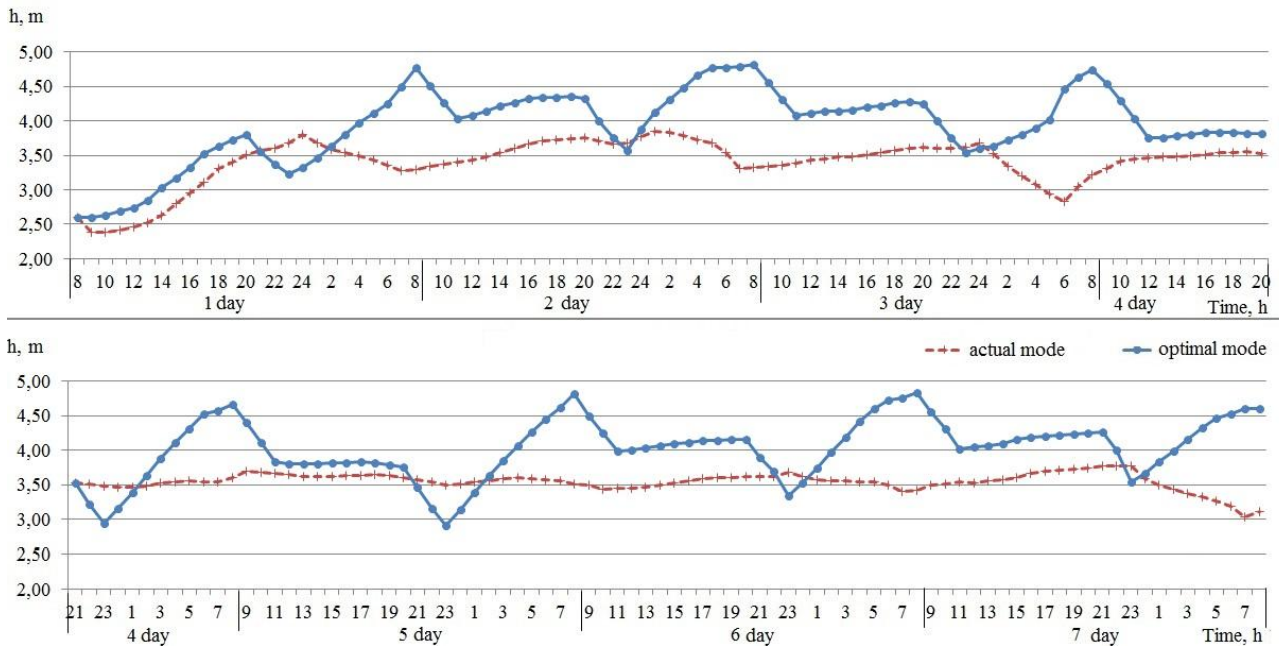


Fig. 7. The change of the estimate of the mathematical expectation of water level in CWR4

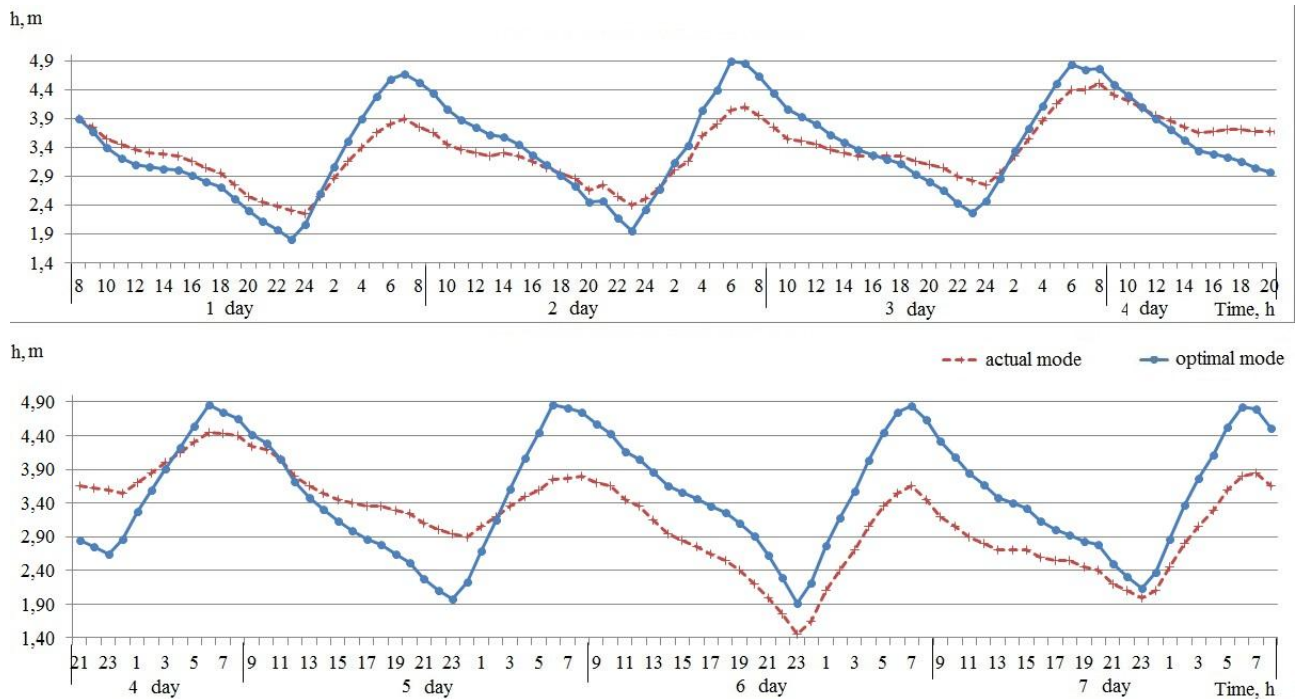


Fig. 8. The change of the estimate of the mathematical expectation of water level in CWR5

From Table. 5, Fig. 7, Fig. 8 it is seen that the capacities CWR4 and CWR5 under optimal mode of operation of WM are used more efficiently, i.e. the water level in the reservoirs varies in a wider range.

CONCLUSIONS

Scientific novelty:

1. a new class of the problems of optimal stochastic control of the operation with discrete-time of the complex dynamic objects, different from well-known by the introduction of additional extreme and probabilistic constraints on the phase variables has been developed;

2. the mathematical formulation of the problems of optimal stochastic control of the modes of operation of WM with extreme and probabilistic constraints on the phase variables has been presented;

3. to obtain an approximate solution of the examined problem a new strategy of optimal stochastic control of the modes of operation of WM has been offered, it takes into account the specific features of WM as a stochastic object of control operating in the stochastic environment, the use of which has permitted to simplify the solution of the examined problem significantly and provided a significant reduction of the financial expenses for the electricity while the transition to a three-tier tariff.

Practical value: the estimates of the effectiveness of the proposed method compared to the current one are given on the example of one of the largest WM of Ukraine. It is shown that the use of the proposed method has allowed to obtain the economic benefit for the cost of the electricity up to 8,82%, which in absolute terms for the examined sample has figured up to the savings of 658 thousand UAH. per month.

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