Choosing of Optimal Cluster Size in WiMax Mesh Network Under Hierarchical Cross-Layer Routing

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Abstract **— A rational compromise between scalability and performance of WiMax mesh wireless network can be reached by hierarchical cross-layer routing. The paper offers appropriate mathematical model in space of state. The model describes dynamics of clustered mesh wireless network and allows to decompose initial control task into routing and slot allocation inside clusters, and routing and slot allocation between clusters. Simulation results show that chosen cluster size affects network productivity and complexity of optimization problem, its optimal value depends on average distance between source and destination, network size and network connectivity.**

Keywords—wireless mesh-network; hierarchical cross-layer routing; time slot allocation; clustering; size of cluster

I. INTRODUCTION

Effectiveness of Wireless Mesh Network (WMN) entirely depends on procedure of resource allocation in it. Independently of type of link resources (time, frequency, or time-frequency blocks) assignment of some amount of resources to some link means using (including) of the link in route. Thus task of link resource allocation (scheduling) is inseparably linked with routing problem, and in order to maximize network productivity both of them should be solved jointly within so called cross-layer routing.

Specificity of WiMax (IEEE 802.16) mesh networks is related to time division multiple access to common frequency band. The standard defines time slot as unit of resources to be allocated at link layer. Thus task of slot allocation and as a result cross-layer routing have integer nature. Except different heuristics, all described in literature approaches to optimal cross-layer routing can be divided into three groups:

- Graph-based approach based on implementation of cross-layer metrics together with shortest path algorithms [1];
- Flow-based formulation of slot allocation and routing as integer linear optimization problem [2];
- Based on mathematical model in space of state and formulation of cross-layer routing task as nonlinear optimization problem [3].

The mathematical model in space of state has set of advantages in comparison with other approaches. There are dynamic nature that allows to adopt to structural and functional (parametrical) changes; ability to find optimal path (paths) which will have residual capacity accurately appropriate to serviced request; ability to control link and buffer resources simultaneously; ability to improve resource utilization and to maximize network productivity; possibility to operate with quality parameters and guarantee quality of service. On the other hand practical implementation of the model is restricted by its high complexity that gives rise to scalability problem.

A rational way to save all of advantages of model in space of state and at same time to solve scalability problem is related to hierarchical control. It assumes dividing initial control task into set of simpler subtasks every from which should be solved independently and the separate solutions should be coordinated in order to improve whole objective function. Implementation of the hierarchical approach requires clustering of WMN' structure and appropriate modification of model in space of state. The paper offers mathematical model for hierarchical cross-layer routing in clustered WiMax mesh network, and focuses on role of cluster size and its influence on network productivity under offered mathematical model.

II. CONCEPT OF HIERARCHICAL CROSS-LAYER ROUTING IN CLUSTERED NETWORK

The main factor that complicates process of optimal slot allocation and as a result implementation of the model in space of state in whole is related to interference effect. The phenomenon doesn't permit to use same slots by stations if distance between them is not more than predefined communication range (as a rule 2-4 hops). At same time slot reuse allows to improve network productivity up 50%. In order to avoid interference and to simplify process of slot allocation we developed following concept of hierarchical cross-layer routing (fig. 1) [4].

First stage is related to clustering of WMN where all stations inside single cluster are interfered to each other and as a result can not to use same time slots simultaneously. The fact simplifies process of slot allocation inside single cluster. Every cluster is assumed to have own pool of slots and some clusters are assumed have same pools. Slot allocation is offered to be realized by coordinated work of control algorithms at upper and low levels. Upper level (UL) control algorithm solves problem of traffic routing and slot allocation between clusters without taking into account detailed cluster structure. But lower level (LL) control algorithm is aimed at slot allocation within single clusters taking into account solutions for transit traffic which is sent down from upper control level. Thus initial task of cross-layer routing is divided in (N_{cl}+1) tasks, one UL task and N_{cl} LL tasks where N_{cl} is number of clusters in WMN.

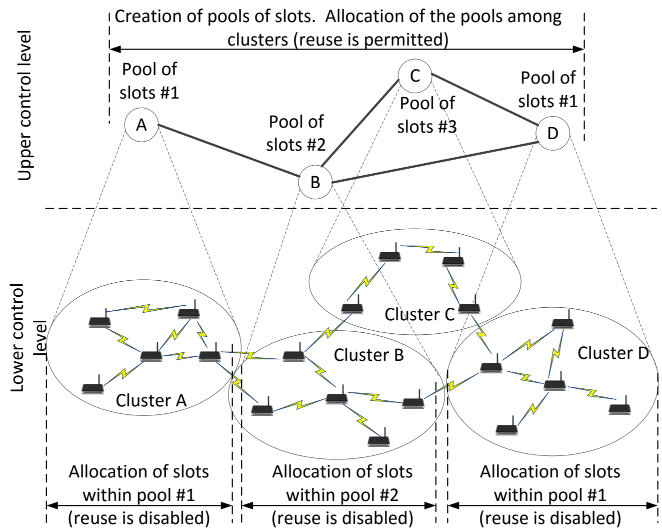

Fig. 1. Concept of hierarchical cross-layer routing in wireless mesh network.

III. MATHEMATICAL MODEL FOR HIERARCHICAL CROSS-LAYER ROUTING IN CLUSTERED WMN

Mathematical description of WiMax mesh network in space of state is based on two main types of variables, which can be monitored and controlled. There are state variable $q_{i.x,j.z}(k)$ representing the data volume that is kept at the instant t_k in buffer of the station $i.x$ and intended for transmission to the station $j.z$ (or queue $(i.x, j.z)$), and binary control variable $\tau_{i.x,j.z}^{r,l.s}(k)$, which equals to 1 if during k-th time interval r-th slot is used in link *(i.x, j.z)* for transmission of flow addressed to station $l.s$. It's assumed that structure of WiMax mesh network is clustered and every mesh-station is identified by hierarchical address $i.x$ where i is cluster' identifier but x reflects sequence number of a mesh station within i-th cluster.

The variables describe process of traffic servicing in WMN, order of slot using and used routes at same time. Mathematically their interrelation can be defined by following system of equalities of states [3, 4]:

$$q_{i.x,j.z}(k+1) = q_{i.x,j.z}(k) - \sum_{\substack{h.s\in S_{i.x}^1,\\ h.s\neq i.x}} \sum_{r\in\Theta_i} m_{i.x,h.s}^{r}(k)\tau_{i.x,h.s}^{r,j.z}(k)n +$$

$$+ \sum_{\substack{g.w\in S_{i.x}^1,\\ g.w\neq i.x,j.z}} \sum_{r\in\Theta_i} m_{g.w,i.x}^{r}(k)\tau_{g.w,i.x}^{r,j.z}(k)n + \xi_{i.x,j.z}(k)\Delta t, \qquad (1)$$

where $i,j=\overline{1,N_{cl}}$, $x=\overline{1,N_v^i}$, $z=\overline{1,N_v^j}$, $j.z\neq i.x$; N_v^i is number of stations in i-th cluster; $k=0,1,2,\ldots$; $\Delta t = t_{k+1}-t_k$ is the sampling interval; $m_{i.x,h.s}^{r}$ is number of bits of the user's data that can be carried by r-th slot in link $(i.x,h.s)$; $S_{i.x}^1$ is a set of distance-1 neighboring stations to the $i.x$-th station; $\xi_{i.x,j.z}(k)$ is the intensity of the data arrival to the $i.x$-th station at the instant of time t_k addressed to the $j.z$-th station; n is the number of the frames transmitted during time Δt, $n=\Delta t/T_F$; T_F is the frame duration; Θ_i is set (pool) of slots assigned for i-th cluster.

The equalities for state variables with same indexes of current cluster and same indexes of destination cluster, for example, $q_{A.1,C.1}(k)$, $q_{A.1,C.2}(k)$, $q_{A.2,C.1}(k)$ and so on, can be aggregated (by summation) into single equality with state variable $q_{A,C}(k)$ belonging to upper control level:

$$q_{i,j}^{UL}(k+1) = q_{i,j}^{UL}(k) - \sum_{\substack{h\in S_i^1,\\ h\neq i}} \sum_{r\in\Theta_i} m_{i,h}^{UL\,r}(k)\tau_{i,h}^{UL\,r,j}(k)n +$$

$$+ \sum_{\substack{g\in S_i^1,\\ g\neq i,j}} \sum_{r\in\Theta_g} m_{g,i}^{UL\,r}(k)\tau_{g,i}^{UL\,r,j}(k)n + \xi_{i,j}^{UL}(k)\Delta t, \qquad (2)$$

where $q_{i,j}^{UL}(k) = \sum_{x=1}^{N_v^i}\sum_{z=1}^{N_v^j} q_{i.x,j.z}(k)$ is state variable of upper control level, it represents the data volume that is kept at the instant t_k in cluster i and intended for transmission to stations in cluster j; $m_{i,h}^{UL\,r}$ is number of bits of the user's data that can be carried by r-th slot in aggregated external link (i,h); $\tau_{i,h}^{UL\,r,j}(k)$ is binary control variable at upper level, it is responsible for allocation of slots over aggregated links between clusters; $S_i^{UL\,1}$ is a set of distance-1 neighboring nodes to the *i*-th node in graph of clusters; $\xi_{i,j}^{UL}(k)$ is the intensity of the aggregated traffic that arrives to the *i*-th cluster at the instant of time t_k addressed to the stations in *j*-th cluster (external traffic), $\xi_{i,j}^{UL}(k) = \sum_{x=1}^{N_v^i}\sum_{z=1}^{N_v^j} \xi_{i.x,j.z}(k)$.

Note that aggregated state variables $q_{i,j}^{UL}(k)$ should to meet constraints:

$$q_{i,j}^{UL}(k) \geq 0\,, \qquad \sum_{\substack{j=1,\\ i\neq j}}^{N_{cl}} q_{i,j}(k) \leq q_i^{UL\,\max}\,, \qquad (3)$$

where $q_i^{UL\,\max}$ is upper boundary of traffic that can be kept within i-th cluster.

Based on (1) equalities of states for stations inside i-th cluster can be written as

$$q_{i.x,i.z}^{LL}(k+1) = q_{i.x,i.z}^{LL}(k) - \sum_{\substack{i.s\in S_{i.x}^1,\\ s\neq x}} \sum_{r\in\Theta_i} m_{i.x,i.s}^{LL\,r}(k)\tau_{i.x,i.s}^{LL\,r,i.z}(k)n +$$

$$+ \sum_{\substack{i.w\in S_{i.x}^1,\\ w\neq x,z}} \sum_{r\in\Theta_i} m_{i.w,i.x}^{LL\,r}(k)\tau_{i.w,i.x}^{LL\,r,i.z}(k)n + +\xi_{i.x,i.z}^{LL\,\Sigma}(k)\Delta t, \qquad (4)$$

where $\tau_{i.x,i.z}^{LL\,r,i,s}(k)$ is binary control variable at lower level, it is responsible for allocation of slots on links inside

i-th cluster; $m_{i.x,i.s}^{LL\,r}(k)$ is number of bits of the user's data that can be carried by r-th slot in internal link $(i.x,i.s)$; $\xi_{i.x,i.z}^{LL\,\Sigma}(k)$ is aggregated traffic that arrives to queue $(i.x,i.z)$ on station $i.x$ at the instant of time t_k addressed to the stations $i.z$ (including external and internal traffic), $\xi_{i.x,i.z}^{LL\,\Sigma}(k)\Delta t = \xi_{i.x,i.z}(k)\Delta t + \xi_{i.x,i.z}^{ext}(k)\Delta t$; $\xi_{i.x,i.z}^{ext}(k)\Delta t$ is amount of external incoming traffic, information about which should be down by upper control level.

Because initial state variables $q_{i.x,j.z}(k)$ reflect amount of data that are kept in buffer of wireless station, the variables are limited from both sides, zero as lower bound and size of buffer as upper bound:

$$q_{i.x,i.z}^{LL}(k) \geq 0\,, \qquad \sum_{\substack{z=1,\\ z\neq x}}^{N_v^i} q_{i.x,i.z}^{LL}(k) \leq q_{i.x}^{LL\,\max}\,, \tag{5}$$

where $q_{i.x}^{LL\,\max}$ is total size of buffer at $i.x$-th mesh station.

According to offered concept of hierarchical cross-layer routing choosing of control variables $\tau_{i,h}^{UL\,r,j}(k)$ and $\tau_{i.x,i.z}^{LL\,r,i,s}(k)$ is guided by two rules. The first low index in the variable indicates cluster to which it belongs. Because every cluster has own pool of slots Θ_i, unit value of binary variable $\tau_{i,h}^{UL\,r,j}(k)$ or $\tau_{i.x,i.z}^{LL\,r,i,s}(k)$ for i-th cluster is allowed if and only if $r \in \Theta_i$. Reuse of slot is possible for noninterfering clusters which have same pools of slots, but every slot in pool can be used just once, or by upper level, or by lower lever. Allocation of pools, which can be obtained after coloring of conflict graph of clusters, guarantees that between colored with same colors clusters interference is absent [4]. It allows to allocate slots in every cluster within its own pool Θ_i, $i=\overline{1,N_{cl}}$, independently without taking into account process of slot allocation in other clusters. Mathematically it can be written as (for every $r \in \Theta_i$, $i=\overline{1,N_{cl}}$)

$$\sum_{\substack{j=1,\\ j\neq i}}^{N_{cl}} \sum_{\substack{l=1,\\ l\neq i}}^{N_{cl}} \tau_{i,j}^{UL\,r,l}(k) + \sum_{x=1}^{N_v^i} \sum_{\substack{z=1,\\ z\neq x}}^{N_v^i} \sum_{\substack{s=1,\\ s\neq x}}^{N_v^i} \tau_{i.x,i.z}^{LL\,r,i,s}(k) \leq 1\,. \tag{6}$$

Thus problem of routing between clusters and allocation of slots on external links can be formalized as [3]

$$\begin{aligned} J^{UL} = \sum_{k=1}^{a} \Big[& \vec{q}^{\,UL\,T}(k) W_q^{UL} \vec{q}^{\,UL}(k) + \vec{\tau}^{\,UL\,T}(k) W_\tau^{UL} \vec{\tau}^{\,UL}(k) - \\ & - \vec{\tau}^{\,UL\,T}(k) W_{reuse}^{UL} \vec{\tau}^{\,UL}(k) \Big] \to \min \end{aligned} \tag{7}$$

subject to (2), (3), (6) under known values of $\tau_{i.x,i.z}^{LL\,r,i,s}(k)$, where a is the number of intervals Δt, for which the control variables should be calculated; $\vec{q}^{\,UL}(k)$ and $\vec{\tau}^{\,UL}(k)$ are vectors of state and control variables at upper control level respectively; W_q^{UL}, W_τ^{UL} are the diagonal weight matrices of buffer and link resources usage at upper level respectively; W_{reuse}^{UL} is the weight matrix presenting a gain at the cost of the slots reuse at upper control level.

In turn problem of routing and slot allocation inside clusters can be formalized as optimization problem [3]

$$\begin{aligned} J_i^{LL} = \sum_{k=1}^{a} \Big[& \vec{q}_i^{\,LL\,T}(k) W_i^{q\,LL} \vec{q}_i^{\,LL}(k) + \\ & + \vec{\tau}_i^{\,LL\,T}(k) W_i^{\tau\,LL} \vec{\tau}_i^{\,LL}(k) \Big] \to \min \end{aligned} \tag{8}$$

subject to (4), (5), (6) under known values of $\tau_{i,h}^{UL\,r,j}(k)$, where $\vec{q}_i^{\,LL}(k)$ and $\vec{\tau}_i^{\,LL}(k)$ are vectors of state and control variables at lower control level respectively; $W_i^{q\,LL}$, $W_i^{\tau\,LL}$ are the diagonal weight matrices of buffer and link resources usage at lower level respectively.

Thus formulated optimization problems (7) and (8) are related to calculation of different control variables under known variables from other level. It allows to solve the problems one after other, by turn.

IV. PERFORMANCE ANALYSIS AND CHOOSING OF OPTIMAL CLUSTER SIZE

Main practical reason of hierarchical routing is improvement of network scalability by reduction of different databases sizes and size of problem to be solved. Under offered two-level mathematical model a gain from reduction in number of control variables can be estimated as

$$G_{\dim} = \left|\vec{\tau}\right| \Bigg/ \left(\sum_{i=1}^{N_{cl}} \left|\vec{\tau}_i^{\,LL}\right| + \left|\vec{\tau}^{\,UL}\right| \right), \tag{9}$$

where $\left|\vec{\tau}\right|$ is size of control vector in no clustered network with elements $\tau_{i.x,j.z}^{r,l.s}(k)$ (1); $\left|\vec{\tau}_i^{\,LL}\right|$ and $\left|\vec{\tau}^{\,UL}\right|$ are sizes of LL and UL control vectors respectively.

As numerical results show (fig. 2 and 3) gain from reduction in number of control variables is growing with increase of network and cluster sizes, and in parvo with growth of chromatic number of WMN’s structure.

At same time expansion of clusters leads to complication of LL problems. Moreover because reuse of slots is disabled within single cluster, expansion of its size doesn’t allow to reach globally maximal network productivity. In order to estimate performance of the time-division based wireless network let us define coefficient of flow per slot:

$$K_{fps} = \left(\frac{\xi}{N_{sl}^{us}} \right) : \left(\frac{\overline{m}}{T_F} \right), \tag{10}$$

where $\overline{m}$ is average number of bits of the user's data that can be carried by single slot.

Non-dimensional coefficient K_{fps} is ratio between intensity of delivered flow ξ and number of slots N_{sl}^{us} used for the purpose; physically it reflects efficiency of slot's utilization and naturally it is growing with slot's reuse. Figure 4 reflects coefficient K_{fps} implemented to the maximum flow (called as normalized maximum flow) that indicates negative effect of cluster size's expansion. Simulation results show K_{fps} goes down with increase of distance between source and destination stations.

Thus expansion of cluster size leads to reduction in number of control variables and to dissipation in slot's utilization at same time. In order to take into account the two processes simultaneously an integral gain factor $K_{fps}G_{\dim}$ was calculated (fig. 5).

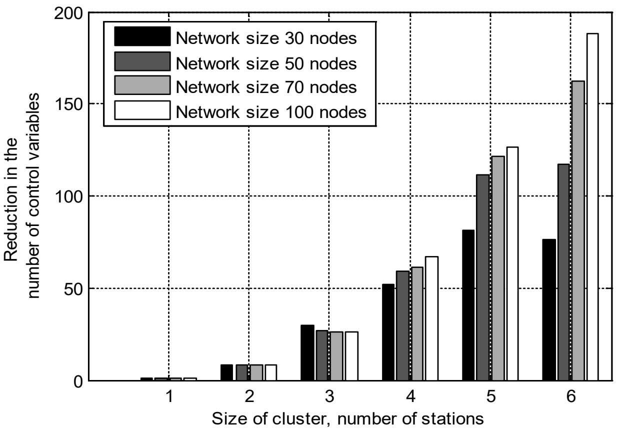

Fig. 2. Dependence of reduction in number of control variables on cluster size and network size (chromatic number is 3).

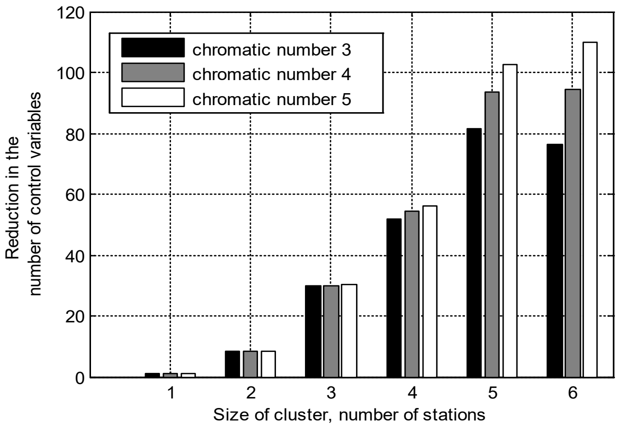

Fig. 3. Dependence of reduction in number of control variables on cluster size and chromatic number (size of network is 30).

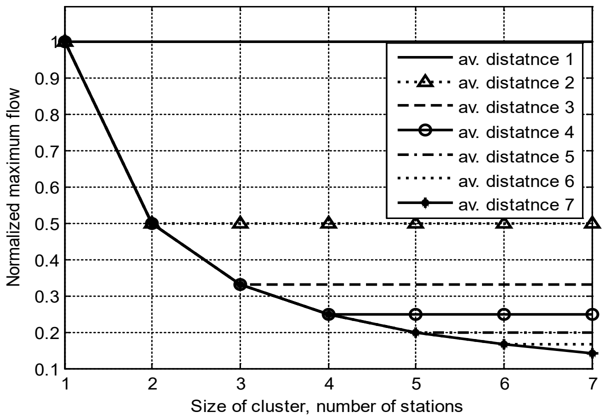

Fig. 4. Dependence of normalized maximum flow on cluster size.

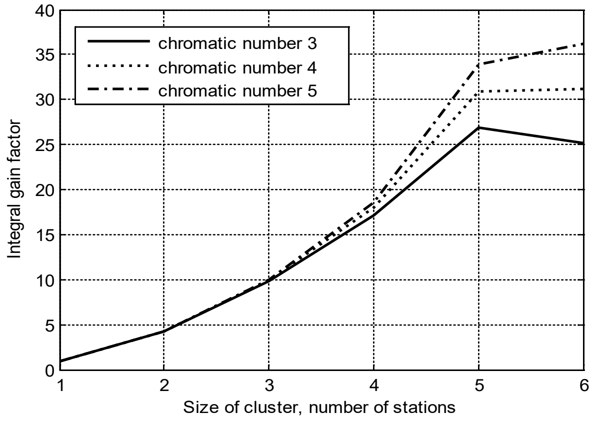

Fig. 5. Dependence of integral factor on cluster size (size of network is 30).

As numerical results show the integral factor has strongly pronounced extremum that corresponds to optimal cluster size. In given example for WMN which contains 30 stations optimal cluster size is 5 (fig. 5). In general case optimal cluster size depends on network size, its connectivity, average distance between source and destination.

V. CONCLUSIONS

Thus offered mathematical model for two-level hierarchical cross-layer routing ensures dynamic nature of control decisions, possibilities to control link and buffer resources simultaneously, allows to solve routing and slot allocation problems jointly, to balance resource using via multipath delivering, to reuse slot. At same time thanks to decomposition of initial task into subtasks the approach solves scalability problem. Simulation results show that cluster has optimal size which depends on network size, its connectivity, average distance between source and destination, and should be defined for every WMN on a case-by-case basis.

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