Descriptor Neural Networks with Arbitrary Characteristic Index

Hahanov V.I., Rutkas A.A.

Abstract – **We consider a difference descriptor system and its modeling with the help of a neural network. The corresponding descriptor network is a special connection of dynamic and static neurons. The network configuration is defined by the Weierstrass's normal form of regular matrix sheaf.**

Keywords – **Descriptor system, neural network, Weierstrass's normal form.**

A descriptor control system is described by differential-algebraic equations, and vector equation of the states of the system contains a singular matrix at the vector of derivatives [1]. A transition from derivatives to finite differences generates a vector difference algebraic equation [2,3]

$$Ax(k+1)+Bx(k)=f_k(x(k)), \quad k=0,1,2,... \tag{1}$$

Here A, B - square ($n \times n$) matrices, and the criterion of the descriptor property of the system is the noninvertibility A ($\det A = 0$). If the characteristic pencil $\lambda A+B$ is regular $\det(\lambda A+B) \not\equiv 0$, then it turns to the normal form of K. Weierstrass [4], and resolvent matrix-function $(\lambda A+B)^{-1}$ exists for large λ and satisfies the power estimate [6]

$$\left\|(\lambda A+B)^{-1}\right\| \le C|\lambda|^{p-1}, |\lambda|>r \tag{2}$$

The minimal integer $p \ge 0$ such that estimate (2) is valid is called *the index* of the matrix pencil $\lambda A+B$. Also we call the integer p the characteristic index of system (1). If the matrix A is invertible, in particular when $A=E$, then the index $p=0$ and system (1) is explicit difference system. Thus, if $p \ge 1$, then system (1) is descriptor.

In [5], there is considered a discrete neural network, which models descriptor system (1) of index $p=1$. Here we construct and analyze an artificial neural network, which is described by equations (1) with arbitrary index $p \ge 1$. It is natural to call the built network the descriptor neural network of index p, (Fig. 1). Its construction is determined by the index p and the normal form of the pencil of matrices $\lambda A+B$. Suppose that the matrices A, B in (1) have the block-diagonal normalized form (compare with [4, 5 and 6])

$$A=\begin{pmatrix} E_m & 0 \\ 0 & H \end{pmatrix}, B=\begin{pmatrix} J & 0 \\ 0 & E_{n-m} \end{pmatrix}, H^p=0 \tag{3}$$

The index p of the pencil $\lambda A+B$ coincides with the nilpotency index of the matrix block H: $H^{p-1} \ne 0$, $H^p=0$. Here E_m designates single ($m \times m$)-matrix, J - any ($m \times m$) matrix. In the general case, the matrix H can be a block-diagonal matrix, containing any amount of nilpotent Jordan cells of sizes $s_k \le p$, so that at least one cell has the maximum size p and $\sum_k s_k = n-m$. To find a model of neural network structure, it is enough to consider the case of $p=n-m$ such that H is a unique nilpotent Jordan cell with size p. In accordance to the approach in the theory of neural networks there are nonlinear vector functions $f_k(x)$ in (1) are chosen in the form $f_k(x)=\Psi(Wx+\Theta(k))$, where elements w_{ik} of the matrix W are interpreted as synaptic weights, components $\Theta_i(k)$ of the vector $\Theta(k)$ - as depositions (external influences at k-th step). In accordance to breaking up of matrices on blocks (3), the vectors x, Ψ have the representations:

$$x=\begin{bmatrix} v \\ h \end{bmatrix}, \Psi=\begin{bmatrix} \varphi \\ \vartheta \end{bmatrix}; v=\begin{bmatrix} x_1 \\ ... \\ x_m \end{bmatrix}, \varphi=\begin{bmatrix} \psi_1 \\ ... \\ \psi_m \end{bmatrix}.$$

Then the vector equations of the states (1) are rewritten in the form:

$$v(k+1)+Jv(k)=\varphi(Wx(k)+\Theta(k)) \tag{4}$$

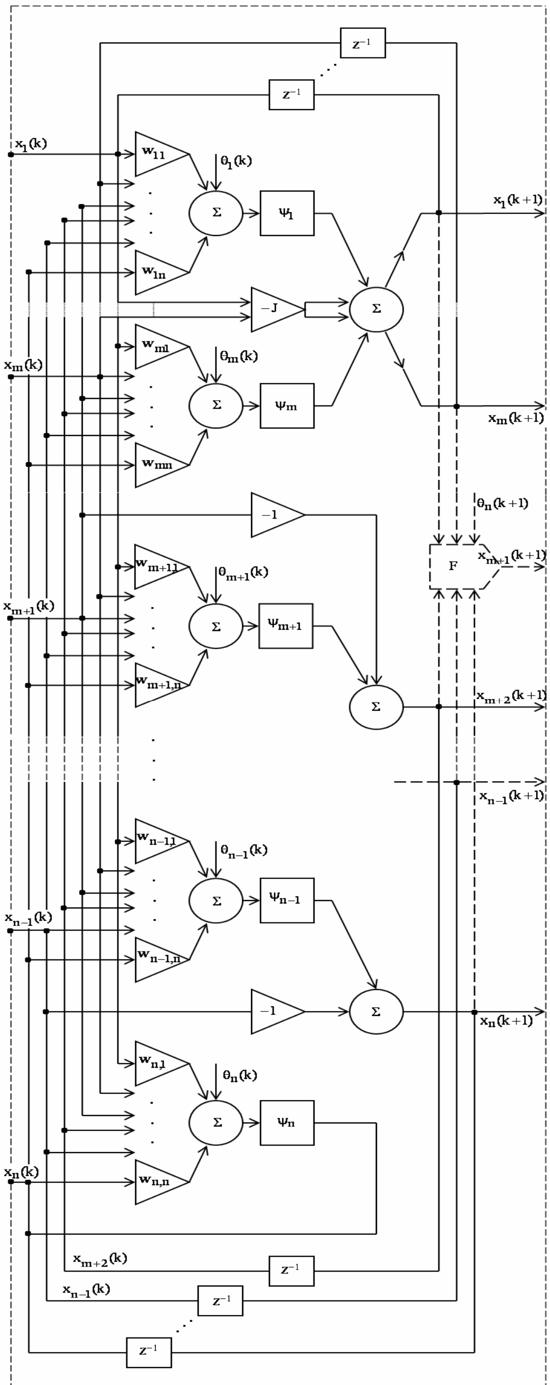
$$\left.\begin{array}{c} x_{m+2}(k+1)+x_{m+1}(k)=\psi_{m+1}(Wx(k)+\Theta(k)) \\ x_{m+3}(k+1)+x_{m+2}(k)=\psi_{m+2}(Wx(k)+\Theta(k)) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ x_n(k+1)+x_{n-1}(k)=\psi_{n-1}(Wx(k)+\Theta(k)) \end{array}\right\} \tag{5}$$

$$x_n(k)=\psi_n(Wx(k)+\Theta(k)) \tag{6}$$

The main dynamic block (4) of m equations can be realized as Hopfield vector dynamic neuron with additional block of multiplying by the matrix ($-J$).

Manuscript received October 30, 2009.
Vladimir Hahanov is with the Kharkov National University of Radioelectronics, Ukraine, 61166, Kharkov, Lenin Prosp., 14, room 321 (corresponding author to provide phone: (057)7021326; fax: (057)7021326; e-mail: hahanov@ kture.kharkov.ua).
Rutkas Andrey is with the Kharkov National University of Radioelectronics, Ukraine, 61166, Kharkov, Lenine Prosp., 14.

In Figure the corresponding dynamic subnet with entrance $v(k)=[x_1(k),...,x_m(k)]^{tr}$ and output $v(k+1)=[x_1(k+1),...,x_m(k+1)]^{tr}$ is represented in the case of *nonmutual* activation functions $\psi_i(u)=\psi_i(u_i)$, $u=Wx+\Theta$, depending on the component u_i of the vector of internal state $u=u(k)$. Therefore, m Hopfield classical dynamical neurons are used in the network realization of equations (4).



Descriptor network of index $p=n-m$

The descriptor part of the neural network in Figure transforms the part $(x_{m+1}(k),...,x_n(k))$ of the entrance vector into the vector $(x_{m+2}(k+1),...,x_n(k+1),0)$, which is a result of the left shift of the vector $(x_{m+1}(k+1),...,x_n(k+1))$. For this purpose, the special connection of $(n-m-1)$ dynamical neurons and McCuloch-Pitts static neuron with activation function ψ_n is used. Static (or algebraic) equation (6) has the following form for ($k+1$)-st step:

$$\psi_n(\sum_{j=1}^{n} w_{n,j}x_j(k+1)+\Theta_n(k+1))-x_n(k+1)=0 \quad (7)$$

Under the conditions

$$w_{n,n+1}\neq 0,\ \frac{d\psi_n(u_n)}{du_n}\neq 0,\ \forall u_n\in R,$$

equation (7) can be explicitly solved in the components

$$x_{m+1}(k+1)=F[(x_1,...,x_m;x_{m+2},...,x_n;\Theta_n)(k+1)] \quad (8)$$

The dotted block F in Fig. 1 corresponds to solution (8). Equations (4),(5),(8) determine the recurrence operator $S_{k+1}(x(k))=x(k+1)$. The operator S_{k+1} depends on the parameters $\Theta(k),\Theta(k+1)$ and it is defined on the manifold $\Lambda_k=\{x(k)\}$ of vectors $x(k)\in R^n$ which satisfy the scalar equation (6).

REFERENCES

[1] Bender D.J., Laub A. *The linear-quadratic optimal regulator for descriptor systems*, IEEE Transactions on Automatic Control. 1987. Vol. AC-32, №6. P.2062-2077.

[2] Bender D.J. Laub A. The linear-quadratic optimal regulator descriptor systems: discrete-time case. Automatica 23 (1987), P.71-85.

[3] Bondarenko M.F., Rutkas A.G. On a class of implicit difference equations, Dopovidi NAN Ukraine. 1998. №7. P. 11-15.

[4] Gantmacher F.R. Theory of matrices. M.: Nauka, 1966. 576p.

[5] Rutkas A. Descriptor Neural Networks And Singular Implicit Dynamic Systems, Proceedings of IEEE East-West Design & Test Synposium (EWDTS'08), Lviv, Ukraine, October 9-12, 2008, P.429-430.

[6] Vlasenko L.A. Evolution models with implicit and degenerate differential equations, Dniepropetrovsk: System technologies, 2006. 273p.