

Metrics of Vector Logic Algebra for Cyber Space

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Abstract - The algebraic structure determining the vector-matrix transformation in the discrete vector Boolean space for the analyzing information based on logical operations on associative data.

Keywords - vector-matrix transformation, discrete vector Boolean space, information analysis.

I. INTRODUCTION

THE purpose of this article is significant decreasing the analysis time of associative data structures through the developing metrics of vector logic algebra for parallel implementation of vector operations on dedicated multiprocessor device. The problems are: 1. Develop a signature, satisfying a system of axioms, identities and laws for the carrier, which is represented by a set of associative vectors of equal length in the logic vector space. 2. Create a signature of the relations for the carrier, represented by a pair: an associative vector – an associative matrix. 3. Develop a signature of the transformations for the carrier, represented by a pair of associative matrices of equal length.

The research subject is the algebraic structures and logic spaces, focused to creating the mathematical foundations of effective parallel computing processes, implemented in a multiprocessor dedicated product.

References: 1. Technologies for parallel computing by dedicated multiprocessor systems [1-2, 10, 11, 15]. 2. Algebraic structures, focused to creating a mathematical apparatus for parallel computing [3-4, 7-10]. 3. Process models for the solving real-world problems on the basis of effective parallel computing [5, 6, 11, 13].

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II. B-METRIC OF THE VECTOR DIMENSION

Vector discrete logic (Boolean) space determines the interaction of objects through the use of three axioms (identity, symmetry and triangle) forming a nonarithmetic B-metric of vector dimension:

$$B = \begin{cases} d(a, b) = a \oplus b = (a_i \oplus b_i), i = \overline{1, n}; \\ d(a, b) = [0 \leftarrow \forall i(d_i = 0)] \leftrightarrow a = b; \\ d(a, b) = d(b, a); \\ d(a, b) \oplus d(b, c) = d(a, c), \\ \oplus = [d(a, b) \wedge \overline{d(b, c)}] \vee [\overline{d(a, b)} \wedge d(b, c)]. \end{cases}$$

Vertices of the transitive triangle (a,b,c) are vectors (Fig. 1), which identify the objects in the n-dimensional Boolean B-Space; the sides of triangle d(a,b), d(b,c), d(a,c) are the distances between vertices, which are also represented by vectors of the length n, where each bit is defined in the same alphabet as the coordinates of the vectors-vertices.

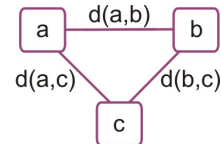


Fig. 1. Triangle of the vector transitive closure

Vector transitive triangle is characterized by complete analogy with the numerical measurement of the distance in the metric M-space, which is specified by the system of axioms, determining the interaction between one, two and three points of any space:

$$M = \begin{cases} d(a, b) = 0 \leftrightarrow a = b; \\ d(a, b) = d(b, a); \\ d(a, b) + d(b, c) \geq d(a, c). \end{cases}$$

The specific of metric triangle axiom lies in numerical (scalar) comparison the distances of three objects, where the interval uncertainty of the result – two sides of a triangle can be greater or equal to a third one – not really suitable for determining the exact length of the last side. Removal of this disadvantage is possible only in a logical vector space, which can form a deterministic view for each characteristic of the process or phenomenon state. Then the numerical uncertainty of the third triangle side in a vector logical space takes the form of the exact binary vector, which characterizes the distance between two objects

and is calculated on the basis of knowledge of the distances for the other two triangle sides:

$$d(a, b) \oplus d(b, c) = d(a, c).$$

The three axioms of the determining a metric are redundant, at least for the vector space, where a single axiom can be used – the interaction between three points: $d(a, b) \oplus d(b, c) \oplus d(a, c) = 0$. Two identities are followed from this law, which determine the relations between one and two points in a space:

$$d(a, b) \oplus d(b, c) \oplus d(a, c) = 0 \rightarrow \begin{cases} d(a, b) = d(b, a) = 0 \rightarrow c = \emptyset; \\ d(a, a) = 0 \rightarrow \{b, c\} = \emptyset. \end{cases}$$

The following fact is interesting. Having regard to the cyclical nature of the triangle, for any two known adjacent (incident) components the third one can be calculated. This concerns both to states (codes) of vertices and to the distances between them:

$$\begin{cases} d(a, b) = d(a, c) \oplus d(b, c) \\ d(b, c) = d(a, b) \oplus d(a, c) \\ d(a, c) = d(a, b) \oplus d(b, c) \end{cases} \quad \begin{cases} d(b, c) = b \oplus c \\ d(a, c) = a \oplus c \\ d(a, b) = a \oplus b \end{cases} \quad \begin{cases} a = d(a, b) \oplus b \\ b = d(b, c) \oplus c \\ c = d(c, a) \oplus a \end{cases}$$

Isomorphism of the set theory concerning the algebra of logic allows determining the vector set-theoretic S-space, where the triangle axiom is defined by symmetric difference Δ , which is analogous to the operation XOR in Boolean algebra:

$$S = \begin{cases} d(a, b) = a \Delta b = (a_i \Delta b_i), i = \overline{1, n}; \\ d(a, b) = [\emptyset \leftarrow \forall i (d_i = \emptyset)] \leftrightarrow a = b; \\ d(a, b) = d(b, a); \\ d(a, b) \Delta d(b, c) = d(a, c), \\ \Delta = [d(a, b) \cap \tilde{d}(b, c)] \cup [\tilde{d}(a, b) \cap d(b, c)]. \end{cases}$$

Here Δ is the symmetric difference operation on the four-digit set-theoretic alphabet $\alpha = \{0, 1, x = \{0, 1\}, \emptyset\}$, represented by the following table:

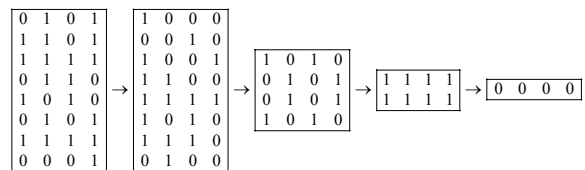
Δ	0	1	x	\emptyset
0	\emptyset	x	1	0
1	x	\emptyset	0	1
x	1	0	\emptyset	x
\emptyset	0	1	x	\emptyset

When determining the distance between two vectors in the S-space the symmetric difference is used, which is isomorphic to the XOR-operation in the Boolean B-space. Examples of calculating the distances between vectors in both spaces (S, B) are given below:

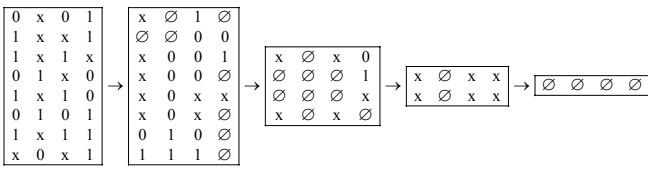
$$S = \begin{array}{c|cccccccc} a & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ b & x & x & 0 & 0 & 1 & 1 & 0 & 0 \\ c & x & x & x & x & 0 & 0 & 0 & 0 \\ \hline d(a, b) & 0 & 1 & \emptyset & \emptyset & \emptyset & x & \emptyset & x \\ d(b, c) & \emptyset & \emptyset & 1 & 1 & x & x & \emptyset & \emptyset \\ d(a, c) & 0 & 1 & 1 & 1 & x & \emptyset & \emptyset & x \end{array}$$

$$B = \begin{array}{c|cccccccc} a & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ b & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ c & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline d(a, b) & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ d(b, c) & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ d(a, c) & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}$$

Vector equal to zero (empty set) for all coordinates means a full match the response and query. As well as the vector equal to 1 (symbol x) for all digits indicating complete contradictoriness the response and query. Number of gradations for a variable can be a finite number that multiple of a power 2 $\alpha = 2^n \rightarrow \{2^2 = 4, 2^4 = 16\}$, which is determined by the power of the Boolean on the universe of n primitive. Otherwise, the symmetric difference can exist only in closed concerning the set-theoretic operations alphabet. Thus, the interaction of two objects in a vector logical space can have either binary or multi-valued deterministic scale of measuring interaction. Hasse diagram of any finite number of primitives (1,2,3,4, ...) can be packed to a variable of logical vector. Moreover, 16 gradations (for instance) of vector interaction by the four primitives exactly indicate not only the degree of proximity by the variable, but in what way they differ – by some primitives, or their combination. Vector operation XOR actually smooths out the changes in the two codes or vectors, that is of interest for the creating digital filters. If it is applied many times, we can get a binary pyramid, where the last vertex is always the zero vector. Thus, the obtained pyramid makes it possible using some redundancy to correct errors in the process of information transferring. The procedure of convolution distances in order to verify the errors of data transferring for the number of vectors equal degree 2 is presented below. 1) Compute all the distances between the binary codes, including the last and first vectors, resulting in a closed geometric figure $c_i = a_i \oplus a_{i+1} (i = n \rightarrow i+1 = 0)$. 2) Compute all distances between non-overlapping pairs of obtained in the first stage codes $c_i = a_{2i-1} \oplus a_{2i} (i = 1, 2, 3, \dots, n)$. 3) Repeat procedure 2 to obtain a package equal to zero in all coordinates. The procedure is illustrated by the following calculations:



Similar actions can be performed and for multivalued vectors, where, for instance, every coordinate is defined in four-digit set-theoretical alphabet, and the procedure is reduced to the obtaining a vector of empty values coordinates:



Here it happens a convolution of a closed space to a single point, Fig. 2, defined in all coordinates by symbols of the empty set, by calculating the distance between vector-objects, and then – the distance between the vector-distances. Otherwise, the modulo sum of all vector-distances, closed in the cycle is equal to an empty vector

$$m_i = c_i \oplus_{j=i+1, n}^{j=i+1, n} c_j \rightarrow m = m_i \oplus_{i=1}^{n-1} m_{i+1}.$$

But this procedure is characterized by less error diagnosis depth – the detecting an incorrect bit is possible. While a binary tree of space convolution makes it possible to increase the diagnosis depth up to a vector pair.

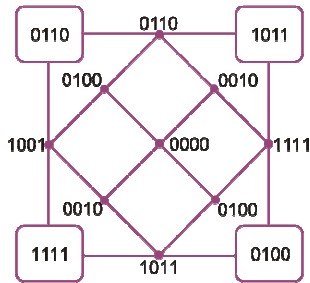


Fig. 2. Closed space convolution

Space convolution is of interest for many real-world problems: 1) Diagnosing and correcting errors when the transmitting information via communication channels. 2) Detecting faults in digital products based on fault detection tables. 3) Searching faults in digital products based on multivalued fault detection tables.

The essence of the space convolution lies in the metric of transitive triangle, which can be transformed by shifting the right side of the equation to the left:

$$d(a, b) \oplus d(b, c) = d(a, c) \rightarrow d(a, b) \oplus d(b, c) \oplus d(a, c) = 0.$$

This definition assign primary importance not elements of the set, but the relations, thereby reducing the system of metric axioms from three to one and to extend its action on an arbitrarily complex structures of n-dimensional space. The classical metric definition for determining the interaction between one, two and three points in a vector logical space is a special case of B-metric when $i = 1, 2, 3$ respectively:

$$M = \begin{cases} d_1 = 0 \leftrightarrow a = b; \\ d_1 \oplus d_2 = 0 \leftrightarrow d(a, b) = d(b, a); \\ d_1 \oplus d_2 \oplus d_3 = 0 \leftrightarrow d(a, b) \oplus d(b, c) = d(a, c). \end{cases}$$

In particular, metric, functional and other kinds of spaces in the sum also give zero. For example, a figure with sides 1, 2, 3,

according to all the textbooks, is not a triangle, because three points are located in line, Fig. 3. But the axiom of metric transitive closure uses a structure consisting of three points on the plane with different coordinates, which is strictly called a triangle. Then a figure with sides 1, 2, 3, according to the definition of the metric, is a triangle with two zero angles and the third one, equal to 180 degrees, where all the conditions for the three sides are met: $a + b \geq c \rightarrow 1 + 2 = 3$.

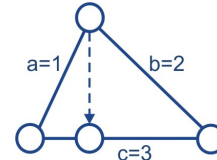


Fig. 3. Metric triangle

III. CONCLUSION

Information vector logic space as a subset of a metric one determines the interaction between a finite numbers of objects by means of the introduced definitions, axioms of identity, symmetry and transitivity of the triangle. At that the last property degenerates into a strict equality, which makes it possible potentially to reduce by a third volume of binary information about the interaction of objects, due to the convolution of any closed logical space in the zero-vector.

Beta-metric of a vector logic space, presented by a zero-sum of cycle distances of binary codes, creates a fundamental basis for all logical and associative problems of synthesis and analysis related to the searching, recognition and decision-making.

Based on the beta metric and the three quality criteria of interaction between vector logical objects in the same space a beta-criterion is created. It makes it possible to determine effectively, accurately and adequately the quality of object interaction, when searching, pattern recognition and decision-making by calculating the xor-function.

Algebra of vector logic creates an infrastructure mathematical service of a vector logical space for the solving real-world problems of synthesis and analysis. It consists of three components: vector, vector-matrix and matrix algebraic structures. Signature of algebras is given by a standard set of logical vector operations AND, OR, NOT, XOR to determine the interaction between compatible objects of a carrier, which form a binary n-dimensional vectors and compatible by the dimension matrix.

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