Spectral Analysis in Problems of Electromagnetic Sources Detection and Multilayer Structures Identification

Andriy Synyavskyy, Volodymyr Antonyuk, Mykhajlo Lobur,Yevgenij Klepfer

Abstract— **The principles of spectral estimation are investigated in this paper to develop a common approach to the solution of both localization problem of spatially-distributed electromagnetic sources and identification problem of multilayer structures. The process of experimental data registration in a linear antenna array and multi-frequency means of multilayer structure non-destructive evaluation is described by proposed identical models. The order and parameters of the models are determined by mean of stochastic approach to the spectral estimation, which allows to obtain higher spectral resolution and accuracy in case of low signal to noise ratio.**

***Index Terms*—Direction of arrival estimation, maximum likelihood, multilayer structure, order of model, spectral analysis.**

I. Introduction

The year 1807 is considered to be the beginning of spectral analysis theory, when Jean Baptiste Joseph Fourier presented an idea of series which was later named after his name - Fourier's series [1]. For over a number of years the spectral analysis has been rapidly developing and penetrating in different fields of pure and applied mathematics. Being a numerical method of the spectral analysis, a fast Fourier's transform essentially increased the efficiency of applied using of the spectral representation. It is well known that discrete Fourier's transformation has limited spectral resolution [2]. The main achievement of modern spectral analysis is to overcome this limit by means of using stochastic approach to the spectral estimation. The idea, which is taken as a principle of high resolution spectral analysis, is to use a certain model of spectrum the parameters and order of which should be estimated on the basis of discrete input data.

Manuscript received November 20, 2008.

A. Synyavskyy is with the Physico-mechanical institute after G.V. Karpenko, National Academy of Sciences of Ukraine (e-mail: a.synyavskyy@gmail.com).

V. Antonyuk is with Lviv Radio Engineering Research Institute (e-mail: avp-lviv@i.ua).

M. Lobur is with Lviv Polytechnic National University (e-mail: mlobur@polynet.lviv.ua).

Ye. Klepfer was from Lviv Radio Engineering Research Institute.

The direction of using spectral analysis theory to the solution of two practical problems is developed in the paper. The first one is the radiation (rediffraction) sources of plane electromagnetic wave detection as well as determination of their angle of arrival. The second problem relates to the identification of the plane dielectric multilayer structures parameters. The aim of this paper is adaptation of stochastic methods of spectral analysis and estimation their consistency in processing of noisy data measurement of scattered electromagnetic field with the purpose of describing the geometry of objects which form this field.

II. The Stochastic Approach to High Resolution Spectral Estimation

The problem of spectral estimation consists in parametric or nonparametric calculation of values of spectral density for each value of spectral parameter. There are many natural processes which are characterized by discrete spectrum that is why there is no need to determine value of spectral density in the fields between spectral lines, namely in these places where this value is equal to zero. The spectral analysis problem in case of the discrete spectrum reduces to estimation of number of spectral lines, the spectral parameters values corresponding to them and the spectral density for each spectral line.

A. Problem formulation of discrete spectrum estimation

It is assumed that random process which consists of a finite number D of harmonic components with frequencies $\omega_1, \omega_2, \omega_3 \cdots \omega_D$ and random component $\mathbf{n}$ is considered as a main model in the theory of spectral estimation [2-7]. One realization of the random process that is observed in N different moments can be presented in discrete form:

$$\mathbf{x} = \mathbf{V}(\mathbf{w}) \cdot \mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{s} \in C^{D \times 1}$ - is a column vector of complex values of either determinate or random modulating signals and $\mathbf{n} \in C^{N \times 1}$ - is a realization of random component determined in the discrete times.

The further analysis is based on an assumption that this random component of the investigated process is the identically-distributed Gaussian value. $\mathbf{V}(\mathbf{w}) \in C^{D\times N}$ - is the Vandermonde matrix with exponential coefficients in the expression (1). This matrix can be presented in parameterized form $\mathbf{V}(\mathbf{w}) = [v(\omega_1) \mid v(\omega_2) \mid \cdots \mid v(\omega_D)]$, or:

$$\mathbf{V}(\mathbf{w}) = \begin{bmatrix} e^{-j\omega_1\Delta} & e^{-j\omega_2\Delta} & \cdots & e^{-j\omega_D\Delta} \\ e^{-j2\omega_1\Delta} & e^{-j2\omega_2\Delta} & \cdots & e^{-j2\omega_D\Delta} \\ \vdots & \vdots & & \vdots \\ e^{-jN\omega_1\Delta} & e^{-jN\omega_2\Delta} & \cdots & e^{-jN\omega_D\Delta} \end{bmatrix},$$

where $\mathbf{w} = [\omega_1 \mid \omega_2 \mid \omega_3 \cdots \omega_D]^T$ - is a vector of value of spectral parameters and $[\ ]^T$ - is the notation of transposed operator; Δ - is the time interval with which the investigated random process realization is sampled.

Depending on the existing a priori data the main problem of spectral analysis is formulated how the estimate of vector $\hat{\mathbf{w}} = [\hat{\omega}_1 \mid \hat{\omega}_2 \mid \hat{\omega}_3 \cdots \hat{\omega}_D]^T$ can be found according to the input data $\mathbf{x}$.

Under some conditions the model order D, determinate values or statistic properties of the vector $\mathbf{s}$ of the modulating signals as well as the parameters of random component $\mathbf{n}$ can be also treated as unknown values.

The statistic theory is based on the principle of expectation of random values according to a set of realizations. Thus, for obtaining more accurate results of spectral estimation one should take into consideration not single realization $\mathbf{x}$ but whole set of such realizations $\mathbf{X} = [\mathbf{x}_1 \mid \mathbf{x}_3 \mid \mathbf{x}_2 \mid \ldots \mid \mathbf{x}_K]$, $\mathbf{X} \in C^{N\times K}$, which are registered under the same conditions of experiment. In this case a number of realizations K means how many similar measurements are made. Each of the matrix vectors $\mathbf{X}$ corresponds to the observation model (1) of random value under investigation:

$\mathbf{x}_k = \mathbf{V}(\mathbf{w})\mathbf{s}_k + \mathbf{n}_k,\ k = 1...K$.

This random component has a noise nature in each experiment and is statistically-independent random value with covariance matrix $E[\mathbf{n}\mathbf{n}^H] = \sigma^2\mathbf{I}$, where σ^2 - is the random component variance and $\mathbf{I} \in R^{N\times N}$ - is the identity matrix; $E[\ \ ]$ - is the expectation operator.

The solution of statistic spectral estimation problem depends on what kind of supposition is made concerning the character of modulating signal vectors $\mathbf{s}$. There are several principal differences between statistic and deterministic definition of these vector parameters. In this paper the main attention is paid to the case where the vector elements s are normally distributed random values with the covariance matrix $E[\mathbf{s}\mathbf{s}^H] = \mathbf{S}$. Thus the rule of linear transformation of normal-distributed random values allows to establish that the vector elements $\mathbf{x}$, the values of which are registered as a result of experiment, are normally-distributed too with their probability density function:

$$\begin{aligned} p(\mathbf{x}_k) &= \frac{1}{\left(\sqrt{2\pi}\right)^N \sqrt{det(\mathbf{B})}} exp\left\{-\frac{1}{2}\mathbf{x}_k^H \mathbf{B}^{-1}\mathbf{x}_k\right\} = \\ &= \frac{1}{\left(\sqrt{2\pi}\right)^N \sqrt{det(\mathbf{B})}} exp\left\{-\frac{1}{2} tr\left(\mathbf{B}^{-1}\mathbf{x}_k\mathbf{x}_k^H\right)\right\} \end{aligned}, \quad (2)$$

where $det(\ )$ and $tr(\ )$ - are notations of determinant and trace of matrix; $[\ ]^H$ - is a complex-conjugate transpose operator, and $\mathbf{B} = E[\mathbf{x}\mathbf{x}^H]$ - is a covariance matrix which can be presented by the sum of two matrixes [7]:

$$\mathbf{B} = \mathbf{V}(\mathbf{w})\mathbf{S}\mathbf{V}^H(\mathbf{w}) + \sigma^2\mathbf{I}. \quad (3)$$

Statistic independence of separate measurements gives possibility to write a joint distribution function of random values $\mathbf{x}_1, \mathbf{x}_3, \mathbf{x}_2, \ldots \mathbf{x}_K$, as the product of distribution density functions (2) for each of the experiment observations:

$$\begin{aligned} p(\mathbf{X}) &= \prod_{k=1}^{K} p(\mathbf{x}_k) = \\ &= \frac{1}{\left(\left(\sqrt{2\pi}\right)^N \sqrt{det(\mathbf{B})}\right)^K} exp\left\{-\frac{1}{2} tr\left(\mathbf{B}^{-1}\sum_{k=1}^{K}\mathbf{x}_k\mathbf{x}_k^H\right)\right\} \end{aligned}. \quad (4)$$

B. Estimation of spectral parameters

If a priori data about solution are unavailable, the problem of statistic estimation can be considered as one of maximization of likelihood function. This likelihood function is written according to the expression (4):

$$\begin{aligned} \ell = ln\, p(\mathbf{X}) &= -\frac{NK}{2} ln(2\pi) - \frac{K}{2} ln(det(\mathbf{B})) - \\ &- \frac{1}{2} tr\left(\mathbf{B}^{-1}\sum_{k=1}^{K}\mathbf{x}_k\mathbf{x}_k^H\right) \end{aligned}. \quad (5)$$

Analysis of the function (5) allows to conclude that having input data $\mathbf{x}_1, \mathbf{x}_3, \mathbf{x}_2, \ldots \mathbf{x}_K$ the estimator of their covariance matrix $\mathbf{B}$ by the maximum likelihood principle can be written as follows:

$$\hat{\mathbf{B}} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}_k\mathbf{x}_k^H. \quad (6)$$

The covariance matrix $\mathbf{B}$ and its estimation $\hat{\mathbf{B}}$, which is obtained by the expression (6), are positively defined Hermitian matrix. This matrix can be factorized using properties of matrix eigenvector decomposition:

$$\mathbf{B} = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^H + \mathbf{U}_n\mathbf{\Lambda}_n\mathbf{U}_n^H \text{ and} \quad (7)$$

$$\hat{\mathbf{B}} = \hat{\mathbf{U}}_s\hat{\mathbf{\Lambda}}_s\hat{\mathbf{U}}_s^H + \hat{\mathbf{U}}_n\hat{\mathbf{\Lambda}}_n\hat{\mathbf{U}}_n^H, \quad (8)$$

where $\mathbf{U}_s$ and $\mathbf{U}_n$ - are the rectangular matrixes ($\mathbf{U}_s \in C^{N\times D}$ and $\mathbf{U}_n \in C^{N\times(N-D)}$), columns of which are eigenvectors that correspond to the useful signal $\mathbf{s}$ and random noise component $\mathbf{n}$; $\mathbf{\Lambda}_s$ and $\mathbf{\Lambda}_n$ - are diagonal matrixes, with eigen values located on their diagonal in a descending order.

The problem of likelihood function maximization (5) can be written for unknown vector values of spectral parameters $\mathbf{w} = [\omega_1 \,|\, \omega_2 \,|\, \omega_3 \cdots \omega_D]^T$. There is no possibility to present this problem solution in an explicit form. Thus instead of direct solution the related functional minimization problem of several variables is considered [3]:

$$\hat{\mathbf{w}}_{MLU} = \underset{\mathbf{w}}{arg\,min}\left(det\left(\mathbf{P}_V \hat{\mathbf{B}} \mathbf{P}_V + \frac{tr\left(\mathbf{P}_V^{\perp}\hat{\mathbf{B}}\right)\mathbf{P}_V^{\perp}}{N-D} \right)\right), \quad (9)$$

where $\mathbf{P}_V = \mathbf{V}(\mathbf{w})\left[\mathbf{V}^H(\mathbf{w})\mathbf{V}(\mathbf{w})\right]^{-1}\mathbf{V}^H(\mathbf{w})$ and $\mathbf{P}_V^{\perp} = \mathbf{I} - \mathbf{P}_V$ are matrix-projectors on orthogonal subspaces.

In case the spectral analysis is based on the assumption about determinate character of the vector elements of modulating signals, the solution of maximization problem [3] of conditional likelihood function can be found from:

$$\hat{\mathbf{w}}_{MLC} = \underset{\mathbf{w}}{arg\,min}\left(tr\left(\mathbf{P}_V^{\perp}\hat{\mathbf{B}}\right)\right). \quad (10)$$

Thus being an unknown value, the modulating signal vector $\mathbf{s}$ can be estimated in an explicit matrix form:

$$\hat{\mathbf{s}} = \left[\mathbf{V}^H(\hat{\mathbf{w}})\mathbf{V}(\hat{\mathbf{w}})\right]^{-1}\mathbf{V}(\hat{\mathbf{w}})\mathbf{X}. \quad (11)$$

According to the expressions (9) and (10), the solution of maximum likelihood problem requires using a numerical approach to the minimization of function of several variables. An ambiguous solution is a characteristic peculiarity of such minimization problems [8]. In other words, global extremum should be found between many local ones. That is why to simplify spectral analysis, the methods [2-6,9], which are asymptotically-equivalent to the maximum likelihood one, are often used on practice because they are less time consumed and have smaller computational complexity. A method for many signals classification MUSIC (MUltiple SIgnal Classification) [9] is one of the simplified methods. This method reduces to determination of an absolute value of signal projection on subspace, which is spanned on eigenvectors of noise component. An estimation of the unknown spectral parameters requires only seeking all extrema of the function of one variable:

$$\hat{\mathbf{w}} = \underset{w}{arg\,min}\, Q_{MUSIC}(\omega), \quad (12)$$

where $Q_{MUSIC}(\omega) = \mathbf{v}^H(\omega)\left(\mathbf{I} - \hat{\mathbf{U}}_s \hat{\mathbf{U}}_s^H\right)\mathbf{v}(\omega)$ - is a function, which characterizes a spectrum of the investigated process. In the neighborhood of values $\omega_1, \omega_2, \omega_3 \cdots \omega_D$, the function $Q_{MUSIC}(\omega)$ takes its local minimum and the spectrum $1/Q_{MUSIC}(\omega)$ as the inverse function tends to its maximum values.

Because of small computational complexity the method MUSIC is used [2,3] as the first approximation for numerical solution of maximization problem of likelihood function (9) or (10). Having possibility to achieve result with a satisfied accuracy, the MUSIC method is able to accomplish the spectral analysis of experimental data under certain conditions.

III. Determination of direction of arrival of plane waves impinging on a linear array

One of the application branches of high resolution spectral analysis is the theory of space-time signal processing and the theory of antenna arrays [3,8,9]. The main problem is to detect sources of electromagnetic radiation in the space, evaluate their coordinates, filter useful signals from these sources in noisy environment and define their parameters. It is well known, the phase methods are able to detect perfectly a separate source of electromagnetic radiation by means of determination of its spatial coordinates. These methods could miss their efficiency of direction-finding in case of presence of several radiation sources in face of an antenna array system. The theory of spectral estimation allows us to solve this problem by the newest theoretically-founded manner [3].

A linear equidistant antenna array on the fig. 1, the signals of its each element are being processed separately, is able to define simultaneously directions of arrival of electromagnetic waves from spatially-distributed sources. The plane waves from these sources propagating in the homogeneous space with wave coefficient $\kappa = 2\pi/\lambda_0$ impinge on the antenna array from different directions $\boldsymbol{\theta} = [\theta_1 \,|\, \theta_2 \,|\, \theta_3 \,|\cdots|\, \theta_D]^T$.

The model of the array element excitation by a set of the plane waves, which come from different directions, can be presented by analogy to the expression (1):

$$\mathbf{x}(t) = \mathbf{V}(\boldsymbol{\theta}) \cdot \mathbf{s}(t) + \mathbf{n}(t). \quad (13)$$

In the case, the realization $\mathbf{x}_k = \left[\mathbf{x}^{(1)}(k\Delta t) \,|\, \mathbf{x}^{(2)}(k\Delta t) \,|\cdots|\, \mathbf{x}^{(N)}(k\Delta t)\right]^T$ of the random process has the meaning of instantaneous values of currents, which are excited in the elements of linear array at k-th moment. The matrix $\mathbf{V}(\boldsymbol{\theta})$ of exponential components consists of the vectors $\mathbf{v}(\theta_d) = \left[e^{-j\kappa h\, sin(\theta_d)} \,|\, e^{-j2\kappa h\, sin(\theta_d)} \,|\cdots|\, e^{-jN\kappa h\, sin(\theta_d)}\right]^T$. Each of them defines signal delays on separate array elements from separate sources. The further explanation will be made under assumption about uniformity of amplitude characteristic of each array elements and an absence of

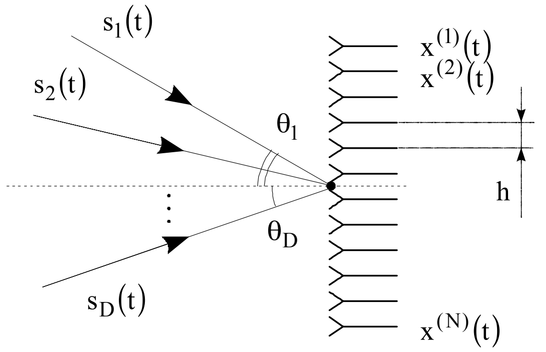


Fig.1. Impinging on a linear array of plane waves from spatially-distributed sources of electromagnetic field

mutual influence between them.

The problem of spectral estimation for the given model of electromagnetic waves registration by the antenna array consists in determination of the impinging angles $\hat{\boldsymbol{\theta}} = \left[\hat{\theta}_1 \,|\, \hat{\theta}_2 \,|\, \hat{\theta}_3 \,|\cdots|\, \hat{\theta}_D\right]^T$ of plane waves coming from the separate sources. These sources can be both statistically-independent and coherent (e.g. $s_k^{(1)} = l \cdot s_k^{(2)}$, where l - is an arbitrary complex coefficient of coherence). The effect of coherence is caused by multipath wave propagation. If there are some coherent waves, which come on array from different directions, the covariance matrix $\hat{\mathbf{B}}$ becomes singular and does not allow described method to separate coherent sources. To avoid this effect an estimation of covariation matrix should be made by a method of spatial smoothing [10]:

$$\hat{\tilde{\mathbf{B}}} = \frac{1}{K(M+1)} \sum_{k=1}^{K} \sum_{p=1}^{N-M} \left[x^{(p)}(k\Delta t) | \cdots \right. \tag{14}$$
$$\left. \cdots | x^{(p+M)}(k\Delta t)\right]^T \left[x^{(p)}(k\Delta t) | \cdots | x^{(p+M)}(k\Delta t)\right]$$

and forward-backward averaging:

$$\hat{\mathbf{B}}_{FB} = \frac{1}{2K} \sum_{k=1}^{K} \mathbf{x}_k \mathbf{x}_k^H + \mathbf{J}\left(\mathbf{x}_k^H \mathbf{x}_k\right)^H \mathbf{J} . \tag{15}$$

where M - is a rank of the result covariance matrix that defines maximal number $(M-1)$ of sources, which can be localized separately; $\mathbf{J}$ - is a counterdiagonal identity matrix.

The method of maximum likelihood (9) or (10) as well as MUSIC method of spectral estimation (12) can be used directly for determination of waves arrival directions on the basis of the estimate of covariance matrix according to the expressions (14) and (15). For example, the result of spectral estimation of simulated data, which are received by an antenna array from four coherent sources, is shown on the fig. 2 for the MUSIC estimation method. As it can be seen from the fig. 2, this method keeps ability to separate closely located coherent sources even if measurements are corrupted by high level noise.

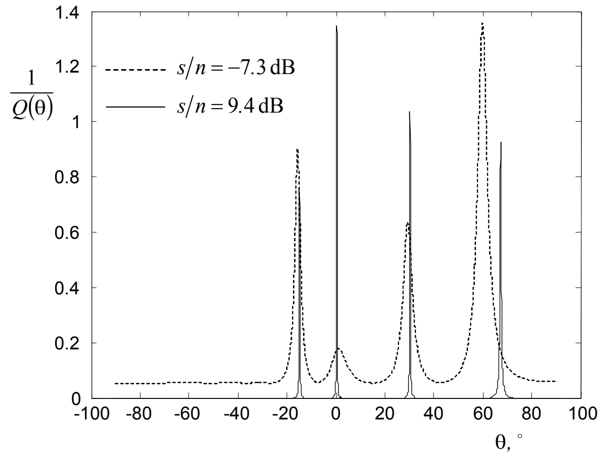


Fig. 2. An example of spectral estimation by MUSIC method for determination of directions of arrival for plane waves from four $D = 4$ coherent sources at $\mathbf{s}^{(d)} = 2$: $\boldsymbol{\theta} = \left[0^\circ \,|\, -15^\circ \,|\, 30^\circ \,|\, 67^\circ\right]^T$, $h = 0.3m$, $\lambda_0 = 1m$, $K = 10$, $N = 30$, $M = 19$.

IV. Identification of multilayer structure on the basis of its scattering matrix analysis

One of the problems of nondestructive diagnostic [11,12] is a determination of multilayer structure parameters on the basis of data about frequency dependence of its scattering matrix coefficients which can be measured by electromagnetic waves sensing. The process of diagnostics of the multilayer structure sample is schematically shown on the fig. 3. The plane wave scattering coefficient $R(\kappa)$ from the surface of multilayer structure and plane wave transmission coefficient $T(\kappa)$ through this structure are established as a ratio of measured electromagnetic field in corresponding points. Usually, registration of these parameters is realized on a grid of discrete frequencies.

The theory of one-dimensional inverse scattering problem [11] gives a rigorous proof of the statement that high-frequency components of reflection $R(\kappa)$ and transmission $T(\kappa)$ coefficients of a structure, which has discontinuities in its material parameters function from the depth y, completely determine the character of these discontinuities as well as parameters of the medium near these discontinuities. On the other hand low-frequency components of reflection coefficients can be used for reconstruction of smooth changes of material parameters function from the depth y [12] in the structure. In case of the investigated structure consists of several homogeneous dielectric layers, the dependence of reflection coefficient and transmission coefficient from frequency (in this case, from the free space wave coefficient $\kappa = 2\pi/\lambda_0$) can be presented in a rational form:

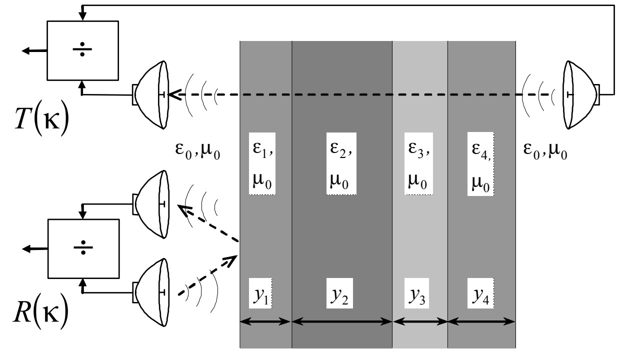


Fig. 3. Determination of electromagnetic waves reflection and transmission coefficients for nondestructive diagnostics of a multilayer dielectric structure

$$T(\kappa)=\frac{1}{\alpha(\kappa)} \text{ and} \tag{16}$$

$$R(\kappa)=-\frac{\beta(\kappa)}{\alpha(\kappa)}, \tag{17}$$

where functions in the nominator and denominator are given by finite sums of exponents:

$$\alpha(\kappa)=\sum_{d=1}^{D} a_d \, exp(-j\kappa z_d) \text{ and} \tag{18}$$

$$\beta(\kappa)=\sum_{d=1}^{D} b_d \, exp(-j\kappa z_d). \tag{19}$$

The coefficients a_d, b_d, z_d are uniquely determined by material parameters $(\varepsilon_1 \ldots \varepsilon_D)$ and thickness $(y_1 \ldots y_D)$ of the multilayer structure layers. Here D value in the expressions (18) and (19) is directly related with the number $P \in N$ of layers in the structure: $D = 2^P$.

Nondestructive diagnostics of multilayer structure can be reduced to the problem [11] of determination of both the coefficients a_d, b_d, z_d and number P of layers. The first step is a measurement recalculation, where two terms in the expressions (16) and (17) are estimated as follows: $\hat{\alpha}(\kappa)=\frac{1}{T(\kappa)}$ and $\hat{\beta}(\kappa)=-\frac{R(\kappa)}{T(\kappa)}$. Secondly, the structure parameter determination problem can be efficiently solved by means of statistic methods of spectral analysis. In order to formalize the problem, a model of input data $\mathbf{x}_{k\alpha}=[\hat{\alpha}_k(\Delta\kappa) | \hat{\alpha}_k(2\Delta\kappa) | \cdots | \hat{\alpha}_k(N\Delta\kappa)]^T$ and $\mathbf{x}_{k\beta}=[\hat{\beta}_k(\Delta\kappa) | \hat{\beta}_k(2\Delta\kappa) | \cdots | \hat{\beta}_k(N\Delta\kappa)]^T$ is written by analogy to the expression (1) in the following form:

$$\mathbf{x}_{k\alpha}=\mathbf{V}(\mathbf{z})\cdot \mathbf{s}_{k\alpha}+\mathbf{n}_k \text{ and,} \tag{20}$$

$$\mathbf{x}_{k\beta}=\mathbf{V}(\mathbf{z})\cdot \mathbf{s}_{k\beta}+\mathbf{n}_k, \tag{21}$$

where $\Delta\kappa$ - is a step of wave coefficient grid, on which the measurements of reflection and transmission coefficients are made; N - is the number of measurements during one experiment; k - is a serial number of an experiment;

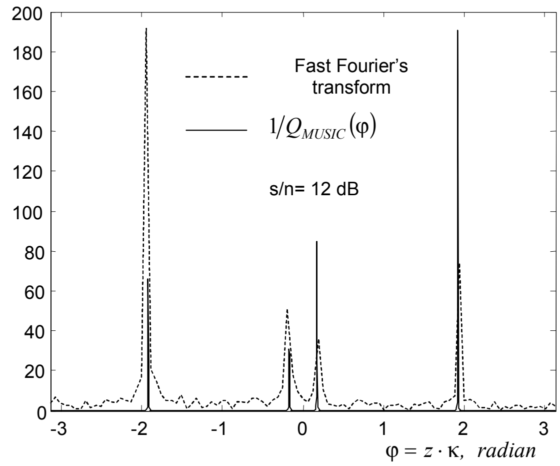


Fig. 4. The spectra of the function $\hat{\alpha}(\kappa)$, which are calculated by means of discrete Fourier's transform and the MUSIC method to detect spectral component in scattering characteristics of a multilayer structure

$\mathbf{z}=[z_1 | z_2 | z_3 | \cdots | z_D]^T$, $\mathbf{s}_{k\alpha}=[\, a_1 | \, a_2 | \cdots | \, a_D \,]^T$ and $\mathbf{s}_{k\beta}=[b_1 | \, b_2 | \cdots | \, b_D \,]^T$ - are the vectors of coefficients, which characterize material parameters of multilayer structure. In the expressions (20) and (21) the matrix of exponential coefficients is formed of D column vectors $\mathbf{v}(z_d)=\left[e^{-j\kappa z_d} | e^{-j2\kappa z_d} | \cdots | e^{-jN\kappa z_d}\right]^T$.

Characterization of the physical meaning of whole coefficients in model (1), which is developed for sensing of multilayer structure by plane electromagnetic waves, allow to estimate the parameters $\hat{\mathbf{z}}=[\hat{z}_1 | \hat{z}_2 | \hat{z}_3 | \cdots | \hat{z}_D]^T$, $\hat{\mathbf{A}}=[\, \hat{a}_1 | \, \hat{a}_2 | \cdots | \, \hat{a}_D \,]^T$ and $\hat{\mathbf{B}}=[\hat{b}_0 | \, \hat{b}_1 | \, \hat{b}_2 | \cdots | \, \hat{b}_D \,]^T$ by any of the presented methods of the spectral analysis. For example, two-layered structure with the parameters $\varepsilon_1 = 50$, $\varepsilon_2 = 10$, $y_1 = 9.4$, $y_2 = 22.1$ has been chosen as a test sample for simulation. Having the simulated values of reflection and transmission coefficient with the signal to noise ratio 12 dB, the spectral parameter $\mathbf{z}$ of reflection characteristics is determined according to the positions of maxima in the spectrum of $\hat{\alpha}(\kappa)$ function. To compare consistency of the method, both spectrum, which is calculated by averaging discrete Fourier's transformations for several result experiments, and spectrum of MUSIC method are shown on the fig. 4 at following parameters: N=100, K=15 and M=40. As one can see on the figure, the method MUSIC allows to achieve higher resolution of spectral estimation in contrast to the Fourier's discrete transformation that indicates to expediency of using stochastic approaches to spectral analysis.

V. Estimation of Model Order in Applications of Spectral Analysis

The main problem of the spectral analysis is the selection of model order, i.e. the number of harmonic components in a process, spectrum of which is analyzed [3]. In the applied problems of interest, the model order (13) corresponds to the number of sources of electromagnetic radiation, the directions of wave arrival from which are estimated. For nondestructive check of multilayer structures by electromagnetic methods, a model order (20-21) and a number of layers of the investigated structure are uniquely related values.

There are several well known approaches to solve the problem of model order selection in spectral analysis. The most frequently used method is based on the Akaike information-theoretical criteria [13] and another one is the method of minimum description length [14]. Describing the investigated processes as a mixture of the Gaussian noise and several harmonious components with random amplitudes, the estimation problem of its model dimensionality can be treated as the problem of simultaneous detection of several signals with the same number. The idea of unifying both problems of detection with constant false alarm rate [15] and spectral analysis for localization of spatially-distributed source as well as multilayer structures identification is developed in this paper.

The statistical representation of registration processes of electromagnetic field parameters in the form of the model (1) allows to write the correlation matrix of input information as the sum (3) of both correlation matrixes of distorted useful signal $(\mathbf{V}(\mathbf{w})\cdot\mathbf{s})$ and noise $\mathbf{n}$. Using orthogonal basis of eigenvectors, the model of the covariance matrix (3) is given in the form:

$$\begin{aligned}\mathbf{B} &= \sum_{i=1}^{D}\left(\lambda_{s_i}+\lambda_{n_i}\right)\mathbf{u}_{s_i}\mathbf{u}_{s_i}^{H} + \sum_{i=D+1}^{N}\lambda_{n_i}\mathbf{u}_{n_i}\mathbf{u}_{n_i}^{H} = \\ &= \sum_{i=1}^{D}\tilde{\lambda}_{s_i}\mathbf{u}_{s_i}\mathbf{u}_{s_i}^{H} + \sigma^2\mathbf{I}\end{aligned}, \quad (22)$$

where $\mathbf{u}_{s_i}$, $i=\overline{1,D}$ and $\mathbf{u}_{n_i}$, $i=\overline{(N-D),N}$ - are eigenvectors of covariance matrix $\mathbf{B}$, which are i-th column of matrix $\mathbf{U}_s$ and $\mathbf{U}_n$, correspondingly; λ_{s_i} and λ_{n_i} - are eigenvalues of matrix $\mathbf{B}$. The same analysis as an eigenvectors decomposition can be made for estimate $\hat{\mathbf{B}}$ of the covariance matrix, which is obtained according to the expression (6):

$$\hat{\mathbf{B}} = \sum_{i=1}^{D}\hat{\tilde{\lambda}}_{s_i}\hat{\mathbf{u}}_{s_i}\hat{\mathbf{u}}_{s_i}^{H} + \sum_{i=D+1}^{N}\hat{\lambda}_{n_i}\hat{\mathbf{u}}_{n_i}\hat{\mathbf{u}}_{n_i}^{H}, \quad (23)$$

where $\hat{\mathbf{u}}_{s_i}$, $\hat{\mathbf{u}}_{n_i}$ and $\hat{\lambda}_{s_i}$, $\hat{\lambda}_{n_i}$ - are the estimate of eigenvectors and eigenvalues of matrix $\hat{\mathbf{B}}$.

Since the covariation matrix estimator are created according to the principle of maximum likelihood, eigenvectors and eigenvalues, which correspond to signal components of the process, should coincide with their estimates $\hat{\mathbf{u}}_{s_i}=\mathbf{u}_{s_i}$ and $\hat{\tilde{\lambda}}_{s_i}=\tilde{\lambda}_{s_i}$ for $i=\overline{1,D}$. Thus, an estimator for variance of noise component can be derived taking into account the equality of the expressions (22) and (23):

$$\hat{\sigma}^2 = \frac{1}{N-D}\sum_{i=D+1}^{N}\hat{\lambda}_{ni}. \quad (24)$$

The difference between models (22) and (23) is used to write the logarithm of likelihood ratio for two events: E_0 - input data corresponds to the model (22), which describes mixture D of harmonic components with Gaussian noise of variance σ^2, and E_1 - input data does not correspond to mixture of a signal with Gaussian noise. The determinant of covariance matrixes [7], which appear in likelihood function (5), can be simply expressed using assumptions about character of these events:

$$det(B_0)=\left(\prod_{i=1}^{d}\hat{\lambda}_i\right)\left(\frac{1}{N-d}\sum_{i=d+1}^{N}\hat{\lambda}_i\right)^{N-d} \text{ and } \quad (25)$$

$$det(B_1)=\left(\prod_{i=1}^{d}\hat{\lambda}_i\right)\left(\prod_{i=d+1}^{N}\hat{\lambda}_i\right). \quad (26)$$

Then the logarithm of likelihood ratio can be rewritten by means of a ratio of the arithmetic mean of least $(N-d)$ eigenvalues of the covariance matrix $\hat{\mathbf{B}}$ to their geometric mean:

$$\begin{aligned} r(d) &= ln\left(\frac{P(E_1)}{P(E_0)}\right)= \\ &= \frac{1}{2}K(N-d)ln\left(\frac{\sum_{i=d+1}^{N}\hat{\lambda}_i}{(N-d)}\left(\prod_{i=d+1}^{N}\hat{\lambda}_i\right)^{-\frac{1}{N-d}}\right). \end{aligned} \quad (27)$$

The obtained expression for logarithm of likelihood ratio is a sufficient statistics for model order d. Determination of model order in the spectral analysis problems can be made by comparison the value $r(d)$ with a fixed threshold. This approach has an essential disadvantage because of ambiguity in selection of the threshold. To eliminate it, the additional constraints [15] are used. Such a formulation of the problem allows to define of constant probability of false alarm P_{FA} as well as to write the problem of model order estimation in a form of one-parameter extremal problem:

$$\hat{D} = arg\,\min_{d}\left(2r(d) + F(d,N,K)\right), \qquad (28)$$

where the functional of constrains $F(d,N,K) = \gamma\left(d\left(N^{\alpha} - d + \beta + \delta + 2\right) + \sigma + 1\right)$ depends on the parameters:

$\alpha = 1 - 0.0001 \cdot 4/(3 + 2K/N)$,

$\beta = (0.06 \cdot N - 0.4) \cdot \ln(2K/N)$,

$\sigma = \sum_{i=1}^{d}\left(i^2/(N^2 - 2) - 0.5\right)$,

$\delta = -0.145N + 4$ and

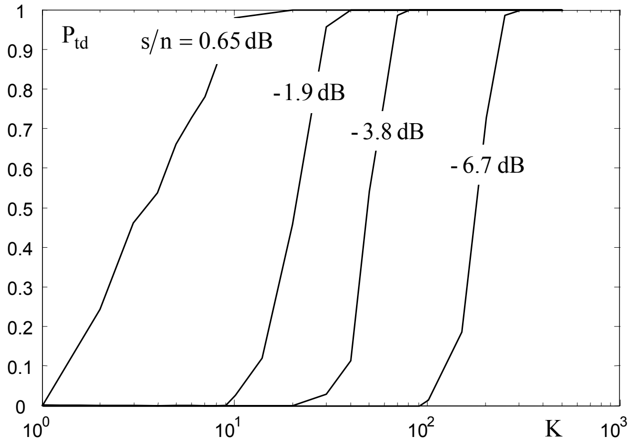
$\gamma = 1 - \frac{4\sqrt{10}}{25\sqrt{N}}\left(\log_{10}(P_{FA}) + 2\right)$.

The statistical modeling is performed to prove the consistency of the developed approach to the estimation of order of model. A case of the plane waves from five $D = 5$ coherent sources of electromagnetic radiation impinging on linear antenna array (the parameters are the same as on the fig. 2) was simulated. The sources are located at different angles $\boldsymbol{\theta} = \left[-15^{\circ}\,|\,0^{\circ}\,|\,30^{\circ}\,|\,45^{\circ}\,|\,67^{\circ}\right]^T$ relatively to the antenna axis. The level of random component on each of the antenna elements was defined according to condition of the experiment. The evaluation $\hat{D}$ of the quantity of radiation sources was made on the basis of the developed method (28). Process of the estimation was simulated repeatedly for fifty times to calculate the value of true detection probability: $P_{td} = \#(D = \hat{D})/50$. The detection characteristics in a form of functions of the true detection probability from realization number K, which is used to estimate the covariation matrix (6), are given on the fig. 5.a for different signal to noise ratio of input data.

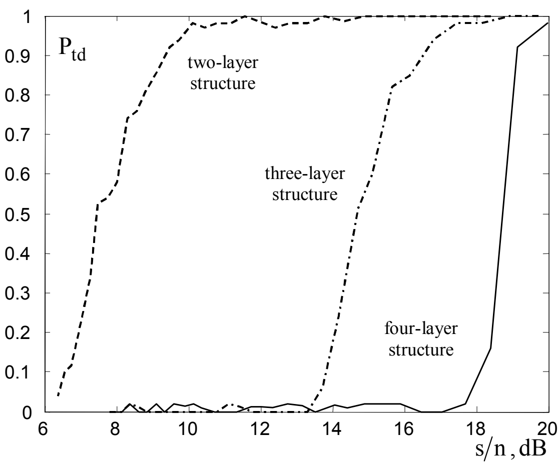
The developed approach is also applied to identify the number of layers in the multilayer structure starting from a frequency dependence of its transmission coefficient. For this purpose a numerical simulation of the electromagnetic waves scattering process for a multilayer structures with different number of layers is carried out. The transmission coefficient in each of the experiments was calculated for $N = 100$ frequency values. The result of estimation of the layers number $\hat{P}$ by means of the developed method (28) is compared to the real number of P layers. The probability of correct identification of the multilayer structures was statistically calculated. Each of the estimation procedures uses the estimate of measurement results covariance matrix, which is found according to the expressions (14) and (15) for $K = 25$ and $M = 40$. The resulted functions of correct identification probability from the signal to noise ratio are given on the fig. 5.b. As it is seen from the figure, a correct identification of structures with a high number of layers requires providing higher signal to noise ratio during measurement of transmission coefficient.

VI. Conclusion

The results of simulation show that discrete Fourier transform as a mean of the processing of noisy experimental data becomes an ineffective because of its limited resolution. On the contrary, the investigated approach to the stochastic solution of the spectral analysis problem allows to obtain a quite accurate estimation of spectral components. While space-time processing on the basis of discrete Fourier transform of received signals by an antenna array does not allow to separate closely-located sources of



a)



b)

Fig. 5. The stochastic characteristics: a) for the estimation of a number of spatially-distributed coherent sources, b) for the estimation of a number of layers in a multilayer structure

electromagnetic radiation, the method MUSIC ensures their separate detection and localization. The similar effect of improvement of efficiency of experimental data spectral analysis was obtained for problem of nondestructive check of multilayer structures by means of the developed approach.

The problem of determination of model order was considered as the separate problem of spectral analysis which provides the consistent estimation the number of spectral components in the investigated process. The measurement errors, which sophisticate the solution of model order determination problem on practice, indicate on an expediency of using stochastically-founded approaches to its solution. The developed approach, which is based on the stochastic solution of the problem by methods of events classification together with imposing constrain, allows to form robust algorithm of the estimation. A small computational complexity of this approach to estimate order of model and a high accuracy of the obtained results points out possibilities to implement this approach in synthesis of antenna array processing algorithm and creation of newest nondestructive means of multilayer structures diagnostic.

REFERENCES

[1] E.A. Robinson, "A historical perspective of spectrum estimation," *Proceedings of the IEEE*, vol. 70, no. 9, pp.885-907, Sept. 1982.

[2] С.Л. Марпл, *Цифровой спектральный анализ и его приложения*. Москва: Мир, 1990.

[3] H.L. Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation*. John Wiley & Sons, Inc., 2002.

[4] P. Stoica, R. Moses, *Spectral Analysis of Signals*. Upper Saddle River, NJ: Prentice Hall, 2005.

[5] S.M. Kay, *Fundamentals of Statistical signal processing: estimation theory*. Upper Saddle River, NJ: Prentice-Hall, 1993.

[6] Y. Hua, A.B. Gershman, Qi Cheng, *High-resolution and robust signal processing*. Basel, NY: Marcel Dekker, Inc., 2004.

[7] Т. Андерсон, *Введение в многомерный статистический анализ*. М.: Физматгиз, 1963.

[8] А.Т. Синявський, В.П. Антонюк, В.Г. Грек, М.В. Лобур, Є.І. Клепфер, "Метод просторової фільтрації сигналу від джерела випромінювання розташованого над розсіюючою поверхнею," *Радіоелектроніка і інформатика*, № 1, с.16-20, 2006.

[9] R.O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. AP-34, pp. 276–280, 1986.

[10] T.J. Shan, M. Wax, T. Kailath, "On spatial smoothing for directional of arrival estimation of coherent signals," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-33, pp. 806–811, 1985.

[11] T. Aktosun, M. Klaus, C. van der Mee, "Inverse wave scattering with discontinuous wavespeed," *Journal of Mathematical Physics*, vol.36, pp.2880-2928, 1995.

[12] J. Modelski, A. Synyavskyy, "A New Numerical Method For Zakharov-Shabat's Inverse Scattering Problem Solution" in *Proc. of XVI Itern. Conference on Microwaves, Radar and Wireless Communications, MIKON'2006*, Krakow, Poland, 2006, vol.1, pp.191-194.

[13] H. Akaike, "A new look at the statistical model identification," *IEEE Trans. Automat. Contr.*, vol. AC-19, pp. 716–723, 1974.

[14] J. Rissanen, "Modeling by the shortest data description," *Automatica*, vol. 14, pp. 465–471, 1978.

[15] J.A. Uber, "*Estimation of the dimensionality of the signal subspace*," A dissertation for the degree of Doctor of Philosophy, Semester 2003, George Mason University, Fairfax, Virginia.

Andriy Synyavskyy received PhD in radio engineering from Lviv Polytechnic National University in 2001. Since 2004 he has been a researcher at Physico-Mechanical Institute of National Academy of Sciences of Ukraine. His research interests are: computer vision, microwave visualization, radar signal and image processing, direct and inverse scattering problems.

Volodymyr Antonyuk graduated from Radio Engineering Faculty in Lviv's Polytechnic Institute in 1972. Now he is in the position of senior researcher at Lviv Radio Engineering Research Institute. He deals with antennas, radar and navigating systems.

Mykhajlo Lobur graduated from Lviv's Polytechnic Institute in 1977. He obtained PhD and DSc in 1986 and 2004, correspondingly. From 1993 to 2001 he was dean of Computer Faculty in Lviv Politechnic National University. Since 2000 he has been in the position of head of CAD department. His research interests include simulation and design of CAD tools for microelectronics.

Yevgenij Klepfer graduated from Taganrog Radio Engineering Institute in 1959. He defended PhD and DSc dissertations in 1967 and 1987, correspondingly. He is a principal researcher at Lviv Radio Engineering Research Institute. His research interests are in signal processing, radar and navigating systems.