Linearized Approach to of Crossed–Field Devices

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Magnetron is one of the first and widespread microwave oscillators where electrons moving in crossed static electrical and magnetic fields and interacted with RF electromagnetic field.

Constructively the modern crossed-field device consists of three parts: cathode, anode structure with cavity resonators and RF energy output [1].

These devices have been analyzed with PIC codes and guiding center theory.

Modeling of crossed–field devices operation related to build and next solve often numerical solution complex of differential equations. Such models need much computer and time resources.

On the other hand any complex system can described simpler equations. Linearized approach is used the linearization of motion equations.

This article aim is used the linearization approach to motion equation of charged particles in crossed electrical and magnetic fields.

In polar coordinates s, φ for electron moving in interaction space motion equations are

$$\begin{cases} \dfrac{d^2 s}{dt^2} - \left(\dfrac{d\varphi}{dt}\right)^2 s = \eta \dfrac{\partial U}{\partial s} - \omega_H s \dfrac{d\varphi}{dt} \\ s\dfrac{d^2\varphi}{dt^2} + 2\dfrac{ds}{dt}\dfrac{d\varphi}{dt} = \eta \dfrac{\partial U}{s\partial\varphi} + \omega_H \dfrac{ds}{dt} \end{cases} \qquad (1)$$

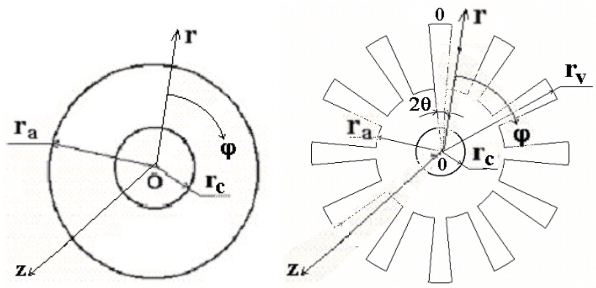
and initial conditions

$$s(0)=1;\ \left.\frac{ds}{dt}\right|_{t=0} = 0;\ \varphi(0)=\varphi_0;\ \left.\frac{d\varphi}{dt}\right|_{t=0} = 0,$$

where $\eta = e/m$; $\omega_H = \eta B$.

We observed and analyzed motion equations for two type of crossed–field devices: magnetron diode and magnetron.

The general view of the interaction space of magnetron diode is shown in figure 1a, and one of magnetron is shown in figure 1b.



a b

Fig. 1. Interaction space

a – magnetron diode; b – magnetron

The potential distribution in interaction space for magnetron diode is described by the following expression

$$U(s) = U_a \frac{\ln s}{\ln s_a},$$

and for magnetron [2, 3]

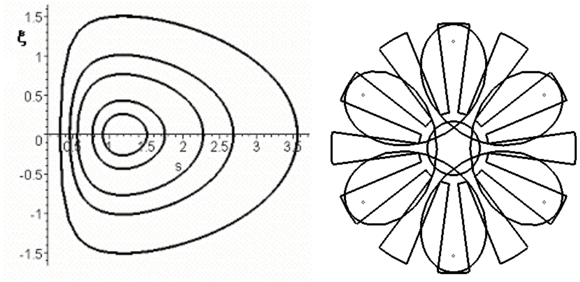
$$U(s,\varphi) = \frac{U_a}{\frac{N\theta}{\pi}\ln\frac{s_v}{s_a} + \ln s_a}\Bigg\{\ln s - 2\ln\frac{s_v}{s_a}\sum_{n=1}^{\infty}\frac{\sin Nn\theta sirs^{Nn}\cos Nn\varphi}{(Nn\theta + \sin 2Nn\theta)\left(sirs_v^{Nn} - sirs_a^{Nn}\right) + \pi sirs_a^{Nn}}\Bigg\}$$

where U_a – anode potential; $s = r/r_c$; $s_a = r_a/r_c$, $s_v = r_v/r_c$; N – number of resonators; 2θ – resonator angle.

To understand the system (1) behavior we must build the characteristic matrix.

The roots of characteristic equation are purely imaginary, so the equilibrium point is the "centre".

Phase portrait of system (1) for magnetron diode is shown in figure 2a, and magnetron – figure 2b.



a b

Fig. 2. Phase portrait of system (1)

a – magnetron diode; b – magnetron

In fig. 2 it shows that the phase trajectories of the dynamical system (1) are closed curves.

It is known if the motion is described by equation (1) is periodic, the phase trajectory corresponding to it will be closed.

Thus, the nonlinear dynamic system (1) has a periodic motion in the radial direction.

Let's simplify the parameter estimation of periodic motion using linearization method.

Considering that in modern magnetron-type devices the relations s < 2 and dimensionless radius s can be represented as $s = 1 + x$. In this cause using linearization method to motion equations (1) these have form

$$\begin{gathered} \frac{d^2x}{dt^2} = \alpha - (1+\alpha)x \\ \frac{d\varphi}{dt} = x \end{gathered}, \qquad (2)$$

where

$\alpha = \dfrac{U_a}{\ln s_a}$ – for magnetron diode;

$\alpha = \dfrac{U_a}{\dfrac{N\theta}{\pi}\ln\dfrac{s_v}{s_a} + \ln s_a}$ – for magnetron.

Solutions of equations (2) can be found in analytical form. Thus we have

$$\begin{gathered} x(t) = \frac{\alpha}{1+b}\left(1 - \cos\sqrt{1+\alpha t}\right) \\ \varphi(t) = \frac{\alpha}{1+\alpha}\left(t - \frac{\sin\sqrt{1+\alpha t}}{\sqrt{1-\alpha}}\right) \end{gathered}. \qquad (3)$$

Expression (1) is parametric form of epicycloids in polar coordinates. Solutions (3) were shown that the charged particle in crossed–fields both magnetron diode and magnetron taken part in two kinds of motion: oscillation motion in radial direction and rotation one in azimuth direction.

The comparison of results (3) and numerical solution (1) which was gotten by computer algebra package Maple™ have maximum error 0,5% for magnetron diode and 2% for magnetron.

It is shown that the use of the linearized approach to study the charged particles' motion in crossed electric and magnetic fields allows to simplify the solution of such problems and to estimate the rotational motion's frequency.

Thus using proposed approach we can investigate analytically behavior of nonlinear dynamical systems "magnetron diode" and "magnetron" and prove existence both oscillation and rotation motion. This is allows to improve upon theory of analytic investigation of crossed–field systems.

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